

1184-27681

Six-Month Progress Report

on

AN ANALYSIS OF WAVE INTERACTIONS IN SWEPT-WING FLOWS

NAG-1-402

by

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MET 37-83-N

July 1984

Abstract

Crossflow instabilities dominate disturbance growth in the leading-edge region of swept wings. It is well known that streamwise vortices in a boundary layer strongly influence the behavior of other disturbances. Amplification of crossflow vortices near the leading edge produces a residual spanwise nonuniformity in the mid-chord regions where Tollmien-Schlichting (T-S) waves are strongly amplified. Should the T-S wave undergo double-exponential growth because of this effect, the usual transition prediction methods would fail. Thus it is important to study interactions of this sort and to develop more realistic criteria for transition prediction.

We modeled analytically the crossflow/Tollmien-Schlichting wave interaction as a secondary instability. The effects of compressibility of the three-dimensional flow over a swept wing will be considered in a future analysis. The effects of suction are included, and different stability criteria are examined. The results are applied to Laminar Flow Control wings characteristic of energy-efficient aircraft designs.

The work is an effort by personnel at Stanford University. At all times the work has been closely coordinated with an experimental program at Virginia Polytechnic Institute and State University (VPI & SU), which is examining the same problem.

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## 1. Introduction

### 1.1 Motivation

Recent research efforts in aircraft design are focused on the idea of Laminar Flow Control (LFC); see, for example, Pfenninger (1977b), Hefner and Bushnell (1977), and Bushnell and Tuttle (1979). The motivation for these efforts has been the realization that a turbulent boundary layer can account for up to 50% of the total drag on a vehicle during cruise. Maintaining a laminar boundary layer by delaying transition to higher Reynolds numbers and therefore increasing fuel efficiency is very promising and of practical importance to the industry.

Transition to turbulence can be caused by many different things Morkovin, 1969, 1977), but in the flight case transition occurs due to the unbounded growth of very small disturbances in the flow. It appears as though the most efficient means for maintaining laminar boundary layers is the use of very weak suction on the wing surface. The weak suction inhibits the growth of unstable disturbances in the boundary layer and delays transition. In the flight case this was demonstrated by Pfenninger and Groth (1961). The recent theoretical work of Reed and Nayfeh (1983) and experimental work of Reynolds and Saric (1983) and Saric and Reed (1983) demonstrate that, in the case of two-dimensional flows, the suction problem is well understood. The suction levels and distributions required to delay transition can be accurately predicted.

Modern LFC transport designs are expected to fly at subsonic speeds near the speed of sound. Their swept wings are being designed with advanced computer codes (Bauer, Garabedian, and Korn, 1972; Kaups and Cebeci, 1977) and have supersonic flow on both the upper and lower surfaces. Typically, the upper surface of the two-dimensional airfoil is characterized by an extensive supersonic flat-pressure region preceded by a leading-edge negative pressure peak and followed by a gradual shock-free recompression to subsonic flow with a subsequent rear pressure rise of the Stratford type (Pfenninger, 1977b); Allison and Dagenhart, 1978; Pfenninger, Reed and Dagenhart, 1980).

Because of wing sweep, the flow over the wing is three-dimensional. The profile of the crossflow velocity component contains an inflection point. This situation is known to be dynamically unstable and creates streamwise

vortices that all rotate in the same sense Gray, 1972; Gregory, Stuart and Walker, 1955; Owen and Randall, 1952). These crossflow vortices are very important and dominate disturbance growth in the leading-edge region (Srokowski and Orszag, 1977). On the other hand, the mid-chord region is dominated by the classical Tollmien-Schlichting instability (T-S waves). Because the flow is locally supersonic, a complete three-dimensional T-S instability must be considered.

One of the major unanswered questions regarding these instabilities concerns the interaction of crossflow vortices and T-S waves. Klebanoff et al. (1962) show that the onset of three-dimensionality is quickly followed by breakdown of the laminar flows. It is well known that streamwise vortices in a boundary layer strongly influence the behavior of other disturbances. Nayfeh (1981) shows that Görtler vortices produce a double-exponential growth of T-S waves. That is, the amplification rate increases exponentially in this case. Herbert and Morkovin (1980) show that the presence of T-S waves also produces a double-exponential growth of Görtler vortices, while Floryan and Saric (1980) show a similar behavior for streamwise vortices interacting with Görtler vortices.

If crossflow vortices and Tollmien-Schlichting waves occur simultaneously, it is possible that premature transition will occur (Pfenninger, 1977a). This possible interaction is especially important on the upper surface of supercritical LFC airfoils in the zero-pressure-gradient region. Vorticity developing in the leading-edge area may linger to the mid-chord region and affect stability there. Amplification of the crossflow vortices near the leading edge produces a residual spanwise nonuniformity in the mid-chord regions, where Tollmien-Schlichting waves are strongly amplified. Should the T-S wave undergo double-exponential growth because of this, the usual transition prediction methods would fail. Thus it is important to study interactions of this sort and to develop more realistic criteria for transition prediction.

## 1.2 Review of Stability Theory

Linear stability theory, or small disturbance theory, in which disturbances are assumed in the form of linear, harmonic, traveling waves, plays an important role in the design of LFC systems. Using it, we can determine the

unstable, or amplifying, range of disturbance frequencies and amplification factors. The  $e^n$ -method (Smith and Gamberoni, 1956; Van Ingen, 1956; Jaffe, Okamura and Smith, 1970) and the modified  $e^n$  and amplitude methods (Mack, 1977) predict trends for changes in the mean flow that delay transition and are therefore very useful. However, they cannot be expected to predict the exact location of transition, because of the strong dependence of transition location on freestream turbulence levels. If the turbulence levels are low, the  $n$ -factor for predicting transition is about 15 (Runyan and George-Falvy, 1979); for higher Reynolds numbers and turbulence levels, an  $n$ -factor of about ten has been suggested (Smith and Gamberoni, 1956; Van Ingen, 1956; Jaffe, Okamura and Smith, 1970). For LFC design, Hefner and Bushnell (1979) suggest a value of five. Saric and Nayfeh (1977) show how these values are changed when the nonparallelism of the basic flow is taken into account.

For parallel, three-dimensional, incompressible flows, Gregory, Stuart and Walker (1955) derive the three-dimensional linear stability equations, including boundary-layer growth and streamline curvature. Then they determine a transformation reducing the three-dimensional temporal problem to a two-dimensional one. For flows over a rotating disk and a sweptback wing, Brown (1961) solves these equations numerically. Nayfeh and Padhye (1980) present a method for calculating neutral stability points for a flat-plate flow. Because calculations of neutral stability (points separating stable and unstable flows) are extremely tedious and difficult in three dimensions, their work is significant. From their iterative scheme, they derive equations relating neutral and nonneutral disturbances. Cebeci and Stewartson (1980) identify an absolute neutral curve, called zarf, for the rotating disk. For given dimensionless frequency  $\omega$ , they find a neutral curve on which the growth rate is zero. That is,  $\alpha_i + \beta_i \tan \phi = 0$ ,  $y = x \tan \phi$ , where  $\alpha_i$  and  $\beta_i$  are the imaginary parts of the complex streamwise and spanwise wavenumbers, respectively, and  $\phi$  is the direction of propagation. They then define the absolute neutral curve zarf on which both  $\alpha_i$  and  $\beta_i$  are zero. Padhye and Nayfeh (1981) do a nonparallel incompressible stability analysis on the X-21 wing and find crossflow vortices to be the most amplified disturbance in regions of pressure gradient. In addition, they find the addition of nonparallelism to give more stable results in the front part of the wing and more unstable results in the rear pressure-rise region.

For parallel, three-dimensional, incompressible stability calculations, a computer code SALLY has been developed by Srokowski and Orszag (1977) that uses the  $e^n$ -method for correlating the transition location. They calculate the maximum temporal amplification rate for a given dimensional frequency at each boundary-layer station from the parallel, incompressible stability equations (the so-called envelope method). Then they use the real part of the group velocity to convert the temporal amplification rate into a spatial one (Gaster, 1962) and integrate along the path defined by the real part of the group velocity. In their limited calculations, they find stationary crossflow vortices to be the most amplified on LFC wings. Mack, in his spatial calculations for the rotating disk (Mack, 1977) and Falkner-Skan-Cooke yawed wedges (Mack, 1978, 1979), also defines the direction of growth as that of the real part of the group velocity. In his work, he introduces his amplitude methods for correlating transition. Cebeci and Stewartson (1980) use the condition that  $d\alpha/d\beta$  be real, a condition also found by Nayfeh (1980a), to calculate an  $n$ -factor for the rotating disk. They start at a point of zero and fix the group-velocity angle as a constant before marching. Nayfeh and Padhye (1979) establish a relation between three-dimensional temporal and spatial stabilities and a relation between spatial stabilities using the complex group velocity. Malik (1980) and Malik and Orszag (1980) compare several methods of transition prediction using incompressible stability theory and conclude that the SALLY code is the most efficient. For the rotating disk, Malik, Wilkinson and Orszag (1981) then use SALLY to calculate temporal eigenvalues, which they convert to spatial eigenvalues using a group-velocity transformation. They then calculate  $n$ -factors using the real part of the group velocity. Dagenhart (1981) considers stationary crossflow vortices and reports that his code MARIA adequately reproduces the stability results of the more complicated stability codes using the same physical disturbance model while using less than 2% of the computer time.

Lekoudis (1979) and Mack (1979) evaluate the effects of compressibility on the stability of the boundary-layer flow over an infinite-span swept wing. They find that for T-S waves the inclusion of compressibility significantly reduces the maximum amplification rate and changes the most unstable wave's orientation. Mack finds for leading-edge-area crossflow that the incompressible theory overpredicts the amplification factor but has little influence on

the wave angle. El-Hady (1980) and Mack (1980, 1981) both report on the parallel compressible stability of the flow over a 23°-swept wing with a supercritical airfoil shape. In their work, Malik and Orszag (1981) describe the computer code COSAL they have developed that efficiently computes temporal eigenvalues by finite differences.

It is apparent that a correct three-dimensional stability analysis must include both compressibility and nonparallelism. Saric and Nayfeh (1975, 1977) show that nonparallelism of the basic flow can be quite important under many circumstances. Nayfeh (1980a) uses the method of multiple scales (e.g., Nayfeh, 1980b) to formulate the three-dimensional problems but presents no numerical results. Nayfeh (1980c) then rigorously formulates the nonparallel 3-D problem. He determines the partial differential equations governing variations of the amplitude and complex wavenumbers and determines conditions on the group-velocity components, making the problem physically realistic. El-Hady (1980) presents some results on a 23°-swept wing with a supercritical airfoil shape. The work of Reed and Nayfeh (1982) represents the state of the art in three-dimensional calculations. Instead of jumping from wave to wave by locally calculating the most amplified disturbance in marching or specifying some artificial condition such as constant spanwise or chordwise wavelength, they formulate the problem by selecting a specific wave at some initial point and then following that one wave along its trajectory, using the group-velocity ratio (Nayfeh, 1980a). A few results for the X-21 wing are given. They find again that the crossflow disturbances are the most amplified in the pressure-gradient region.

If crossflow vortices and Tollmien-Schlichting waves occur simultaneously, it is possible that premature transition will occur (Pfenninger, 1977a). This possible interaction is especially important on the upper surface of supercritical LFC airfoils in the zero-pressure-gradient region. Vorticity developing in the leading-edge area may linger to the mid-chord region and affect stability there. Amplification of the crossflow vortices near the leading edge produces a residual spanwise nonuniformity in the mid-chord regions, where Tollmien-Schlichting waves are strongly amplified. Thus it is important to study interactions of this sort.

For one to use the  $e^n$ -method for the prediction of the transition location, it is necessary to know not only the respective growth rates but the

manner in which disturbances propagate in this three-dimensional interacting flow. Unfortunately, there is a dearth of analytical modeling and experimental data concerning the interaction problem.

Two theoretical approaches are discussed in the next section to model the crossflow/T-S interaction (characteristic of swept wings) as a secondary instability. The first involves the weakly nonlinear combination of crossflow vortices with various oblique-wave solutions to the linear disturbance equations. Craik's resonant triads (Craik, 1971) provide the foundation for this type of approach. The Craik model was considered for the onset of three-dimensionality of T-S waves in boundary layers. These types of models provide only limited understanding to interaction problems, because they specify one specific wavelength for secondary disturbances for a given frequency, providing a very limited window. The secondary disturbances have been observed experimentally in a broad band of wavelengths (Klebanoff, 1962, Hama, 1960, Kachanov and Levchenko, 1982, Thomas and Saric, 1981, Saric and Thomas, 1983, Kozlov, Levchenko, and Saric, 1983). Moreover, interactions have been observed in flows where a Craik-type mechanism is invalid, e.g., subharmonic disturbances in plane Poiseuille flow (Kozlov, 1982).

The second approach described in the next section considers the T-S wave growth as a secondary instability in the presence of finite-amplitude cross-flow vortices. Herbert (1983) has successfully predicted the onset of three-dimensionality in plane Poiseuille flow and in flat-plate boundary layers using such an approach. Herbert considers the secondary instability of modulated finite-amplitude, two-dimensional T-S waves with respect to three-dimensional disturbances. For equilibrium plane Poiseuille flow, he solves a Hill-type system of equations with periodic coefficients and finds the different types of three-dimensional instability associated with different resonance conditions. His results (showing frequency and the broad spectrum of possible wavelengths as a function of amplitude and Reynolds number) compare very well with experimental observations.

Boundary layers are more complicated, because they grow and do not exhibit the cross-stream symmetry of Poiseuille flow. Therefore, this analysis does not apply directly without some approximations. Herbert (1984) uses a shape assumption in which he neglects the nonlinear distortion of the two-dimensional T-S waves at finite amplitude and also ignores the weak variation

of T-S wave amplitude on a viscous time scale. That is, he holds amplitude constant for his analysis so that his basic state (the flat-plate flow with two-dimensional T-S waves superposed) is periodic and a Hill-type system can then be solved for the interaction problem. The results of his parametrical excitation are consistent with available experimental data.

### 1.3 Objectives

It is proposed to model analytically the crossflow/Tollmien-Schlichting wave interactions that are characteristic of flows over swept wings. This modeling is described in the next section. The effects of compressibility and nonparallelism of the basic flow will be included in a future analysis. The effects of suction in reducing the growth of these disturbances will be investigated. Since the flow is complicated by the presence of crossflow vortices, the group-velocity-ratio criterion (Nayfeh, 1980a, 1980c) may not be the only applicable criterion. Thus other criteria will be eventually examined.

## 2. Description of the Analysis

### 2.1 Work in Progress--Analysis

We consider a three-dimensional, incompressible stability analysis. For leading-edge-area crossflow, Mack (1979) finds that the incompressible theory overpredicts the amplification factor but has little influence on the wave angle.

#### A. Basic State

The spatial stability of the basic state (comprised of a three-dimensional boundary-layer flow over a convex or flat surface with a superposed flow corresponding to steady streamwise vortices rotating in the same sense) is examined. This mean flow is typical of the upper surface of a transonic swept wing considered for Laminar Flow Control (LFC) in the flat-pressure region. This region is preceded by a leading-edge negative pressure peak where crossflow vortex instability is expected to be important. These vortices may linger to the zero-pressure-gradient region and affect stability.

The three-dimensional boundary-layer flow is provided by the code of Kaups and Cebeci (1977), a code that solves the laminar compressible boundary-layer equations for a tapered, swept wing with wall mass transfer under the conical-flow assumptions. That is, the pressure gradient along the wing generators is assumed zero, and twist and tip and wing-body effects must be neglected. The governing equations are converted to a two-dimensional form by similarity transformations and then solved numerically by Keller's box method.

Initially, we analyze the form of the solutions of the steady vortex flow. This was done by solutions of the disturbance equations for the vortices. That is, we assume total flow quantities of the form

$$\hat{q}(x,y,z,t) = Q_s(x,y,z) + q(x,y,z) \quad (1)$$

Here,  $Q_s(x,y,z)$  stands for the undisturbed three-dimensional flow provided by the Kaups-Cebeci code, and  $q(x,y,z)$  stands for the small, steady disturbance due to the presence of the crossflow. Substituting the total flow quantities into the Navier-Stokes equations, subtracting the undisturbed part of this basic state (that is, the flow without the vortices), and linearizing, we find the first-order, nondimensional disturbance equations. We define the Reynolds number by

$$R = U_e^* \delta^* / v_e^* \quad (2)$$

Here,  $U_e^*$ ,  $v_e^*$ , and  $\delta^*$  are the dimensional edge freestream chordwise velocity, kinematic viscosity, and displacement thickness, respectively.

We confine our analysis to undisturbed flows that are only slightly non-parallel; that is, the normal velocity component  $v_s$  is small compared with the other components  $U_s$  and  $W_s$ . This in turn implies that all the undisturbed variables must be weak functions of the streamwise and spanwise coordinates compared with the normal coordinate. In other words,

$$U_s = U_s(x_1, y, z_1) , \quad W_s = W_s(x_1, y, z_1) \quad (3a)$$

$$v_s = \epsilon v_s^*(x_1, y, z_1) , \quad v_s^* = O(1) \quad (3b)$$

We describe the relatively slow variations of the undisturbed quantities in the streamwise and spanwise directions by the slow scales  $x_1 = \epsilon x$  and  $z_1 = \epsilon z$ , respectively. Here  $\epsilon = 1/R$ .

We describe the crossflow disturbance quantities in the form

$$q(x, y, z; \epsilon, \epsilon_v) = \epsilon_v [q_0(x_1, y, z_1) \exp(i\theta) + \text{c.c.}] \quad (4)$$

where c.c. stands for complex conjugate and

$$\frac{\partial \theta}{\partial x} = \alpha_v(x_1, z_1) , \quad \frac{\partial \theta}{\partial z} = \beta_v(x_1, z_1) \quad (5)$$

Here,  $\epsilon_v$  describes the magnitude of the crossflow disturbance. The eigenfunctions  $q_0$  are normalized so that their maximum value is one half. The quantities  $\alpha_v$  and  $\beta_v$  are the complex dimensionless quasi-parallel components of the wavenumber in the chordwise and spanwise directions, respectively. Assuming the phase  $\theta$  to be continuously differentiable, we have

$$\frac{\partial \alpha_v}{\partial z_1} = \frac{\partial \beta_v}{\partial x_1} \quad (6)$$

For crossflow vortices, the wavenumber direction  $\psi$  is deflected approximately  $90^\circ$  from the potential-flow direction.

If we adopt the convention

$$z_{01} = u_0, \quad z_{02} = Du_0, \quad z_{03} = v_0, \quad z_{04} = p_0, \quad z_{05} = w_0, \quad z_{06} = Dw_0 \quad (7)$$

we find that the equations governing the eigenfunctions of the crossflow disturbances can be written as a system of six linear first-order ordinary differential equations of the form

$$Dz_{on} - \sum_{m=1}^6 a_{nm} z_{om} = 0 \quad (8)$$

for  $n = 1, 2, \dots, 6$ . The  $a_{nm}$  are the elements of a  $6 \times 6$  variable-coefficient matrix whose nonzero elements are given in Appendix A. The boundary conditions for (8) become

$$z_{01} = z_{03} = z_{05} = 0 \quad \text{at } y = 0 \quad (9a)$$

$$z_{01}, z_{03}, z_{05} \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (9b)$$

The system (8)-(9) constitutes an eigenvalue problem. Given the Reynolds number  $R$  and undisturbed mean-flow profiles, we determine the wavenumbers numerically. Since our problem is linear, it has six linearly independent solutions. The general solution is then a linear combination of these linearly independent solutions, where the coefficients are determined from the boundary conditions. The procedure we use is to guess the eigenvalues and then numerically integrate the system (8) from the boundary conditions for  $y \rightarrow \infty$  to the wall at  $y = 0$ . In our analysis we employ the SUPORT code (Scott and Watts, 1977) and integrate with a Runge-Kutta-Fehlburg scheme using Gram-Schmidt orthonormalization to keep the solution vectors linearly independent. In general, our guess of the eigenvalues is wrong, and therefore one of the boundary conditions at  $y = 0$  is left unsatisfied. We then use a Newton-Raphson scheme on this unsatisfied boundary condition to iterate and try to converge to the correct eigenvalue. The other conditions we satisfy are that the growth rate is a maximum and  $\omega_\beta/\omega_\alpha$  is real.

Once the correct eigenvalues are determined, we recover the eigensolutions and then construct the appropriate basic state for the interaction problem.

## B. Crossflow/T-S Interactions Formulation

In this section, two approaches are discussed to model the crossflow/T-S interaction as a secondary instability. The first approach models the T-S wave growth as a weakly nonlinear system and is similar to the work of Craik. The second approach considers the T-S wave growth as a secondary instability in the presence of finite-amplitude crossflow vortices. Herbert has successfully predicted the onset of three-dimensionality in plane Poiseuille flow and in flat-plate boundary layers using this approach, known as parametric resonance.

In both approaches, we superpose onto the basic state (that is, the three-dimensional boundary-layer flow over a convex or flat surface with a superposed flow corresponding to steady streamwise vortices, rotating in the same sense) two oblique, traveling, harmonic Tollmien-Schlichting waves. To this end, we assume small, unsteady perturbation quantities of the form

$$\varepsilon_T \hat{q}(x, y, z, t) \quad (10)$$

so that, considering (1), (3), and (4), the total flow quantities become

$$\begin{aligned} \hat{q}(x, y, z, t) = & Q_s(x_1, y, z_1) + \varepsilon_v (q_o(x_1, y, z_1) \exp(i\theta) + \text{c.c.}) \\ & + \varepsilon_T (\hat{q}_1(x, y, z, t) + \hat{q}_2(x, y, z, t)) \end{aligned} \quad (11)$$

The small dimensionless quantity  $\varepsilon_T$  represents the order of the amplitude of the two oblique Tollmien-Schlichting waves. Two oblique T-S waves are considered to achieve a resonant or near-resonant condition. The Tollmien-Schlichting instability is assumed to be a secondary instability so that  $\varepsilon_T$  is much smaller than  $\varepsilon_v$  and  $\varepsilon$ . In other words, the terms of order  $\varepsilon_T^2$  are negligible in comparison with  $\varepsilon_T \varepsilon_v$ , and  $\varepsilon_T \varepsilon$ .

Then, using the method of multiple scales (e.g., Nayfeh, 1980b), we look for a uniform expansion of the  $\hat{q}_i$ -quantities ( $i = 1, 2$ ) in the form

$$\hat{q}_i(x, y, z, t) = [\hat{q}_{i0}(x_1, y, z_1, t_1) + \varepsilon \hat{q}_{i1}(x_1, y, z_1, t_1) + \dots] \exp(i\theta_i) \quad (12)$$

where

$$\frac{\partial \theta_i}{\partial x} = \alpha_i(x_1, z_1) \quad (13a)$$

$$\frac{\partial \theta_i}{\partial z} = \beta_i(x_1, z_1) \quad (13b)$$

$$\frac{\partial \theta_i}{\partial t} = -\omega \quad (13c)$$

The quantities  $\alpha_i$ ,  $\beta_i$ , and  $\omega$  are the streamwise wavenumber, spanwise wavenumber, and frequency of the two Tollmien-Schlichting waves, respectively. We substitute these quantities into the Navier-Stokes equations and solve the resulting eigenvalue problem.

#### Infinitesimal Crossflow Disturbance

For growing boundary layers with infinitesimal crossflow disturbances, the parameter  $\epsilon_v = 0(\epsilon)$  to account for both the effect of the vortices and the growth of the boundary layer at the same level. The disturbance equations describing the  $0(\epsilon^0)$  problem and the homogeneous terms of the  $0(\epsilon^1)$  problem are identical in form to the crossflow disturbance equations given by (8)-(9). (The basic state is again the three-dimensional boundary-layer flow over the wing without vortices.) Because of this, a solution for the inhomogeneous  $0(\epsilon^1)$  problem exists only if a solvability condition is satisfied. The inhomogeneous terms must be orthogonal to every solution of the adjoint homogeneous problem. These solvability conditions depend on whether

$$\alpha_1 \approx \alpha_v + \alpha_2 \quad \text{and} \quad \beta_1 \approx \beta_v + \beta_2 \quad (14)$$

or not. If these resonance conditions are not satisfied, the solvability conditions yield two completely uncoupled equations describing the effect of non-parallelism on the two different amplitudes of the two T-S waves. If (14) is satisfied, we introduce detuning parameters  $\sigma_1$  and  $\sigma_2$  defined by

$$\begin{aligned} \alpha_v &= \alpha_1 - \alpha_2 + \epsilon\sigma_1 \\ \beta_v &= \beta_1 - \beta_2 + \epsilon\sigma_2 \end{aligned} \quad (15)$$

and find the following coupled equations describing the amplitudes  $A_1(x_1, z_1, t_1)$  and  $A_2(x_1, z_1, t_1)$  of the two T-S waves

$$g_{11} \frac{\partial A_1}{\partial t} + g_{12} \frac{\partial A_1}{\partial x_1} + g_{13} \frac{\partial A_1}{\partial z_1} = h_1 A_1 + h_{12} A_2 \exp(-i\sigma_1 x_1 - i\sigma_2 z_1) \quad (16a)$$

$$g_{21} \frac{\partial A_2}{\partial t} + g_{22} \frac{\partial A_2}{\partial x_1} + g_{23} \frac{\partial A_2}{\partial z_1} = h_2 A_2 + h_{21} A_1 \exp(-i\sigma_1 x_1 - i\sigma_2 z_1) \quad (16b)$$

The resonance condition produces secular terms and causes the coupling of the amplitudes, leading to possible magnified growth of the T-S waves beyond that predicted by straight linear theory. The coefficients  $g_{ij}$ ,  $h_1$ ,  $h_{12}$ , and  $h_{21}$  are given in Appendix B. The quantities  $h_1$  and  $h_2$  account for the nonparallelism of the mean flow, while  $h_{12}$  and  $h_{21}$  account for the presence of the crossflow vortices.

Considering single-frequency, perfectly tuned disturbances, this system becomes:

$$\frac{\partial A_1}{\partial x} + \frac{\omega_\beta}{\omega_\alpha} \frac{\partial A_1}{\partial z} = \hat{h}_1 A_1 + \hat{h}_{12} A_2$$

$$\frac{\partial A_2}{\partial x} + \frac{\omega_\beta}{\omega_\alpha} \frac{\partial A_2}{\partial z} = \hat{h}_{21} A_1 + \hat{h}_2 A_2 \quad (17)$$

where  $(\omega_\beta/\omega_\alpha)_1$  and  $(\omega_\beta/\omega_\alpha)_2$  are the group velocity ratios of the two T-S waves. This system has the solution:

$$A_1 = \hat{A}_1 e^{S_1 x + S_2 z}, \quad A_2 = \hat{A}_2 e^{S_1 x + S_2 z} \quad (18)$$

locally, where the quantities  $\hat{A}_1$  and  $\hat{A}_2$  are the amplitudes of the two T-S waves as observed at the point  $(x, z)$  and

$$S_1 = \left[ \frac{\omega_\beta}{\omega_\alpha} \frac{1}{2} - \frac{\omega_\beta}{\omega_\alpha} \frac{1}{1} \right]^{-1} \left[ (\hat{h}_1 + \hat{h}_{12} \hat{A}_2 / \hat{A}_1) \frac{\omega_\beta}{\omega_\alpha} \frac{1}{2} - (\hat{h}_2 + \hat{h}_{21} \hat{A}_1 / \hat{A}_2) \frac{\omega_\beta}{\omega_\alpha} \frac{1}{1} \right] \quad (19a)$$

$$S_2 = \left[ \frac{\omega_\beta}{\omega_\alpha} \frac{1}{1} - \frac{\omega_\beta}{\omega_\alpha} \frac{1}{2} \right]^{-1} \left[ (\hat{h}_1 - \hat{h}_2) + \hat{h}_{12} \hat{A}_2 / \hat{A}_1 - \hat{h}_{21} \hat{A}_1 / \hat{A}_2 \right] \quad (19b)$$

The quantities  $S_1$  and  $S_2$  are the growth rates associated with nonparallelism and the presence of the vortices. The total growth rates for the two waves are therefore

$$\sigma_i = -\text{Imag}(\alpha_i) - \frac{\omega_\beta}{\omega_\alpha} \text{Imag}(\beta_i) + S_1 + \frac{\omega_\beta}{\omega_\alpha} S_2 \quad (20)$$

Under these resonance conditions, we shall follow the T-S wave along its trajectory to ascertain its stability characteristics. Since the flow is complicated by the presence of streamwise vortices, marching along the characteristics defined by the group-velocity ratio may not be the only applicable criterion.

#### Finite-Amplitude Crossflow Disturbance

For finite-amplitude crossflow disturbances present in the growing boundary layer, we now assume that  $0(\epsilon) < 0(\epsilon_y) < 0(1)$ . Following the work of Herbert, the mean flow or basic state is described by:

$$Q_o(x, y, z, t) = Q_s(x, y, z) + \epsilon_y [q_o(x_1, y, z_1) \exp(i\theta) + cc] \quad (21)$$

where the eigenfunctions  $q_o$  have been normalized so that

$$\max_{0 \leq y < \infty} |q_o| = 1/2 \quad (22)$$

This expression describes a three-dimensional growing boundary-layer flow over a wing with embedded streamwise crossflow vortices.

We neglect the nonlinear distortion of the crossflow vortices at finite amplitude, ignore the weak variation of crossflow wave amplitude on a viscous time scale, and apply Floquet theory. Under these conditions and considering the condition  $\alpha_1 \approx \alpha_v + \alpha_2$  and  $\beta_1 \approx \beta_v + \beta_2$  to achieve resonance, the first-order disturbance equations describing the two superposed oblique T-S waves are

$$i\alpha_1 u_1 + Dv_1 + i\beta_1 w_1 = 0 \quad (23a)$$

$$i\alpha_2 u_2 + Dv_2 + i\beta_2 w_2 = 0 \quad (23b)$$

$$\begin{aligned} & \left( -i\omega + i\alpha_1 U_s + i\beta_1 W_s + \frac{\alpha_1^2 + \beta_1^2}{R} \right) u_1 + DU_s v_1 \\ & + i\alpha_1 p_1 - \frac{1}{R} D^2 u_1 + i\epsilon_y \left\{ \left[ \left[ \alpha_2 + \alpha_v \right] u_o \right. \right. \\ & \left. \left. + \beta_2 w_o \right] u_2 + \beta_v u_o w_2 \right\} + \epsilon_v v_o D u_2 \\ & + \epsilon_v D u_o v_2 = 0 \end{aligned} \quad (24a)$$

$$\begin{aligned}
& \left( -i\omega + i\alpha_2 U_s + i\beta_2 W_s + \frac{\alpha_2^2 + \beta_2^2}{R} \right) u_2 + D U_s v_2 \\
& + i\alpha_2 p_2 - \frac{1}{R} D^2 u_2 + i\epsilon_V \left\{ \left[ \alpha_1 - \bar{\alpha}_V \right] \bar{u}_o \right. \\
& \left. + \beta_1 \bar{w}_o \right\} u_1 - \bar{\beta}_V \bar{u}_o w_1 \} + \epsilon_V \bar{v}_o D u_1 \\
& + \epsilon_V D \bar{u}_o v_1 = 0
\end{aligned} \tag{24b}$$

$$\begin{aligned}
& \left( -i\omega + i\alpha_1 U_s + i\beta_1 W_s + \frac{\alpha_1^2 + \beta_1^2}{R} \right) v_1 + D p_1 \\
& - \frac{1}{R} D^2 v_1 + i\epsilon_V \left( \left[ \alpha_2 u_o + \beta_2 w_o \right] v_2 \right. \\
& \left. + \left[ \alpha_V u_2 + \beta_V w_2 \right] v_o \right) + \epsilon_V v_o D v_2 \\
& + \epsilon_V D v_o v_2 = 0
\end{aligned} \tag{25a}$$

$$\begin{aligned}
& \left( -i\omega + i\alpha_2 U_s + i\beta_2 W_s + \frac{\alpha_2^2 + \beta_2^2}{R} \right) v_2 + D p_2 \\
& - \frac{1}{R} D^2 v_2 + i\epsilon_V \left( \left[ \alpha_1 \bar{u}_o + \beta_1 \bar{w}_o \right] v_1 \right. \\
& \left. - \left[ \bar{\alpha}_V u_1 + \bar{\beta}_V w_1 \right] \bar{v}_o \right) + \epsilon_V \bar{v}_o D v_1 \\
& + \epsilon_V D \bar{v}_o v_1 = 0
\end{aligned} \tag{25b}$$

$$\begin{aligned}
& \left( -i\omega + i\alpha_1 U_s + i\beta_1 W_s + \frac{\alpha_1^2 + \beta_1^2}{R} \right) w_1 + D W_s v_1 \\
& + i\beta_1 p_1 - \frac{1}{R} D^2 w_1 + i\epsilon_V \left( \alpha_V w_o u_2 \right. \\
& \left. + \left\{ \left[ \beta_2 + \beta_V \right] w_o + \alpha_2 u_o \right\} w_2 \right) \\
& + \epsilon_V v_o D w_2 + \epsilon_V D w_o v_2 = 0
\end{aligned} \tag{26a}$$

$$\begin{aligned}
& \left( -i\omega + i\alpha_2 U_s + i\beta_2 W_s + \frac{\alpha_2^2 + \beta_2^2}{R} \right) w_2 + D W_s v_2 \\
& + i\beta_2 p_2 - \frac{1}{R} D^2 w_2 + i\epsilon_V \left( -\bar{\alpha}_V \bar{w}_o u_1 \right. \\
& \left. + \left\{ \left[ \beta_1 - \bar{\beta}_V \right] \bar{w}_o + \alpha_1 \bar{u}_o \right\} w_1 \right) \\
& + \epsilon_V \bar{v}_o D w_1 + \epsilon_V \bar{D} w_o v_1 = 0
\end{aligned} \tag{26b}$$

The symbol  $\bar{\phantom{w}}$  indicates the complex conjugate. This coupled system of disturbance equations is numerically solved by a spectral collocation method using Chebyshev polynomials. We apply an exponential mapping (Spalart, 1984)

$$y = -\eta_o \ln \eta \tag{27}$$

that transforms  $y = [0, \infty)$  to  $\eta = [1, 0]$ . The value of  $\eta_o$  governs the concentration of points near the wall. Only odd Chebyshev polynomials are used in this formulation so that the infinite boundary conditions are automatically satisfied (Herbert, 1984). In this analysis, we use thirty polynomials giving a  $123 \times 123$  real system of algebraic equations. The results are observed to be strongly dependent on the accuracy of resolution of the basic state.

The spatial solution of this system provides the dispersion relation

$$\omega = \omega(\alpha, \beta, R, \epsilon_V) \tag{28}$$

The growth rates of the two T-S waves are therefore

$$\sigma_i = -\text{Imag}(\alpha_i) - \frac{\omega \beta}{\omega \alpha_i} \text{Imag}(\beta_i), \quad i = 1, 2 \tag{29}$$

Again we shall follow the T-S wave along its trajectory to ascertain its stability characteristics. The amplification factor  $n$  is then the integral along the trajectory of the growth rate.

## 2.2 Work in Progress-Calculations

At present the interaction codes are being developed using X-21 wing data. This wing, derived from the NACA 65A210 airfoil, has leading and trailing edge sweepback angles of  $33.2^\circ$  and  $19.1^\circ$ , respectively. The  $x, y, z$

coordinate system used in the stability analysis is shown in Fig. 1. The input upper surface pressure, suction, and calculated displacement thickness Reynolds number distributions are shown plotted against percent chord in Figs. 2, 3, and 4, respectively. These quantities apply for a freestream speed of 774.4 feet per second and a 14.66-foot chord.

#### A. Crossflow

The crossflow instability is expected to dominate in the pressure-gradient regions. The crossflow/T-S interaction we examine will occur at the leading-edge negative pressure-gradient region. Amplification of crossflow disturbances there produces a residual spanwise nonuniformity that could linger to regions where T-S waves are strongly amplified. For this reason, we focus our attention on the leading-edge area.

Figure 5 shows typical streamwise and crossflow velocity profile distributions for the leading-edge area; these are computed at 1.5% chord. In this plot,

- $U_s^*$  = dimensional chordwise speed
- $U_e^*$  = dimensional chordwise edge speed
- $W_s^*$  = dimensional spanwise speed
- $W_e^*$  = dimensional spanwise edge speed

Note the inflection point in the crossflow profile, a situation known to be dynamically unstable. Also of interest is the relatively small amplitude of the crossflow velocity in comparison with the streamwise velocity. The distribution of maximum (in magnitude) dimensional crossflow speed  $W_{\max}$  versus percent chord is shown in Fig. 6; the values are overall very small.

Two dimensionless parameters commonly used both in experiment and theory to correlate crossflow-vortex appearance with freestream and boundary-layer conditions are the local crossflow Reynolds number

$$Re_{CV} = \frac{W_{\max} y_{10}}{v_e^*}$$

and local shape factor

$$H_{CV} = \frac{y_{\max}}{y_{10}}$$

Distributions of these plotted versus percent chord are shown in Figs. 7 and 8, respectively. Vortices appear when crossflow Reynolds numbers are on the order of 100 and shape factors are on the order of 0.3; these figures are consistent with our findings. Here  $y_{\max}$  is the local normal-direction location of  $W_{\max}$  and  $y_{10}$  is the normal-direction location above  $y_{\max}$  where the crossflow speed is 10% of  $W_{\max}$ .

A spatial stability analysis on this flow yields the following results: Figure 9 shows the amplitude distributions of two perpendicular disturbance profiles, one in the direction of the wave angle, the other normal to this direction. These are computed at 1.5% chord. The larger amplitude portion lies in the direction perpendicular to the wave and is normalized to 1/2 to give proper meaning when an amplitude  $\epsilon_y$  is introduced.

Figure 10 shows parallel and nonparallel calculations for the most unstable disturbance at the leading edge. For Padhye and Nayfeh (1981) nonparallelism is a stabilizing influence. Here we see that nonparallelism is initially stabilizing, then destabilizing. Figures 11, 12 show how that disturbance's wave angle and group velocity vary with percent chord. It is apparent that the most unstable disturbance lies in the direction perpendicular to the freestream and is a crossflow-type growth. The group-velocity direction is almost the same as the potential-flow direction.

#### B. Crossflow/T-S Interaction--Infinitesimal Disturbance

An appropriate basic state is formed and the interaction problems now considered. Because the Craik-type approach is expected to provide only limited insight into the interaction problem, only a few results are presented here to demonstrate the model. The model depends on knowledge of the initial conditions (that is, the amplitude ratio) of the two interacting T-S waves, as well as the amplitude of the crossflow disturbance. Moreover, the model provides a single disturbance wavelength for a given frequency.

Figure 13 shows growth rate (defined by Eq. (20)) plotted versus T-S wave amplitude ratio for three different crossflow velocity magnitudes (0%, 0.6%, 1.2%) for a chord location of 0.5% and a frequency of 325 Hz. As is evident from the plot, the stability of the oblique T-S wave is strongly dependent on the amplitude ratio. Predicting this ratio for an experiment may be difficult, and this model is therefore impractical.

### C. Crossflow/T-S Interaction--Finite Amplitude

For finite-amplitude disturbances, we use a Herbert-type approach to model the interaction. Numerical solutions of Eqs. (23)-(26) provide growth rate (Eq. (29)), wavenumber, and group velocity-ratio distributions as a function of crossflow vortex amplitude. For a chord location of 1.5% and a frequency of 325 Hz, these are shown in Figs. 14-16, respectively. The presence of crossflow vortices, even with small amplitude, apparently causes magnified growth of the oblique T-S waves. The spanwise nonuniformity modulates the growth rates. In addition, a broad band of wavelengths can be observed as a function of amplitude. This is consistent with experimental observations for other three-dimensional interactions--specifically, the occurrence of three-dimensionality in boundary-layer flows. Figure 16 shows the variation of group velocity ratio with amplitude.

For a chord location of 2.5%, Figures 17 and 18 show growth rate and dimensionless wavenumber as functions of frequency and crossflow vortex amplitude. Higher-frequency disturbances appear to be more affected by the presence of the spanwise nonuniformity, as evidenced by the magnified growth rates in Fig. 17. The wavenumber behaves qualitatively the same for the two frequencies.

Figures 19 and 20 show growth rates and amplification factors for a T-S wave of frequency 325 Hz and initial amplitude of 0.6%. Results show parallel, zero-amplitude and finite-amplitude rates. The growth rates are integrated numerically in marching along the trajectory of the wave to find amplitude ratio. Figure 20 compares a parallel, zero-amplitude with a finite-amplitude model. The results clearly show double-exponential growth for the finite-amplitude analysis as compared with the zero-amplitude approach. It is clear that the usual transition prediction methods will fail in such a flow; premature transition will occur.

#### 2.3 Remarks

The analysis will not be restricted just to the effects of crossflow on the T-S instability. The role of T-S waves in affecting the growth of crossflow instabilities will also be examined. Under certain conditions, crossflow vortices can also be amplified in the midchord region of hybrid laminar flow

airfoils. In this case, we assume the crossflow as the secondary instability and proceed with an approach analogous to that above.

At this time, there is no suitable criterion for establishing transition when both crossflow and T-S waves are present. For the lack of anything better, a linear relationship is assumed between the n-factors for crossflow transition ( $n_{CF}$ ) and T-S transition ( $n_{TS}$ ). Usually one assumes

$$n_{TS} = 12 - (1.2) n_{CF}$$

with some error bands, as a transition criterion. The present research will be directed toward better understanding of the relationship between  $n_{TS}$  and  $n_{CF}$ .

The effectiveness of suction will be examined. As was shown by Lekoudis (1978) with wall suction, if the percentage of permeable area is small, if most of the flow there is directed normal to the surface, if the perforations are closely spaced, and if the surface has small permeability, then it is acceptable to specify zero boundary conditions for the disturbances. In our case, this means that both the crossflow vortices and the T-S waves satisfy no-slip and no-penetration. Suction will therefore be introduced into the undisturbed mean flow from the Kaups Cebeci code. The stability results of this modification of the undisturbed mean flow will be applied to typical LFC situations. It is expected that a new capability for analyzing the stability of compressible swept-wing flows will result. Whenever appropriate, the work will be applied to the presently ongoing transonic LFC experiments being conducted at NASA-Langley Research Center in the eight-foot Transonic Pressure Tunnel (Harvey and Bobbitt, 1982).

### 3. Personnel

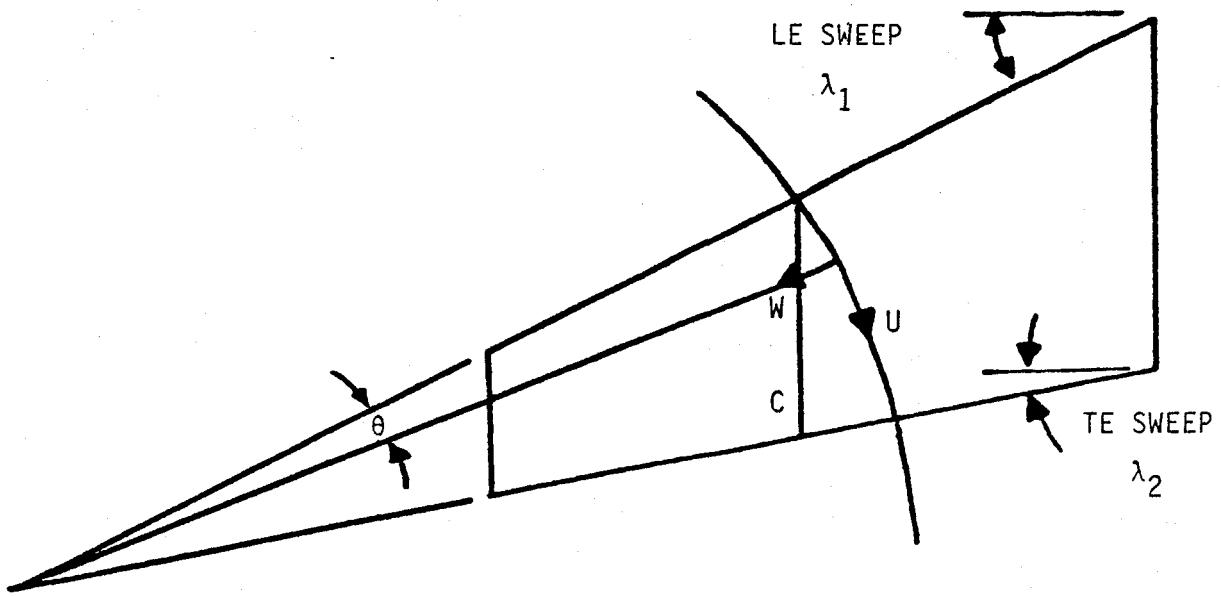
The principal investigator for this project is Helen L. Reed, Assistant Professor of Mechanical Engineering, Stanford University. William S. Saric, Professor of Engineering Science and Mechanics, VPI & SU, has been available to consult on this program. The detailed resume of Dr. Reed is contained in Appendix C.

Helen Reed received her Ph.D. in Engineering Mechanics in 1981 from VPI & SU and joined the faculty at Stanford University in September, 1982. Before coming to VPI & SU, she worked with W. Pfenninger at NASA-Langley on problems of supercritical airfoils and laminar flow control. In addition to her innovative modeling of the suction-strip problem in her dissertation (Reed and Nayfeh, 1983), she has developed the state-of-the-art three-dimensional, compressible stability code (Reed and Nayfeh, 1982) which will be used in the proposed work.

William Saric received his Ph.D. in Mechanics in 1968 from the Illinois Institute of Technology. He worked at Sandia Laboratories for ten years in the Reentry Vehicle Division and the Atomic and Fluid Physics Division, and he has been at VPI & SU for the past eight years. For the past seven years, he has been conducting theoretical and experimental studies of boundary-layer stability problems for LFC applications (Nayfeh and Saric, 1981; Reynolds and Saric, 1982). He is principal investigator on an experimental program of 3-D wave interaction in boundary layers that is directly related to the proposed work.

The author is a member of the Heat Transfer and Turbulence Mechanics (HTTM) group in the Department of Mechanical Engineering at Stanford, which consists of eight faculty members, one research associate, and approximately 35 Ph.D. candidates. The other members of the HTTM group will also play important roles as consultants.

4. Figures



**Fig. 1. X-21 wing coordinate system.**

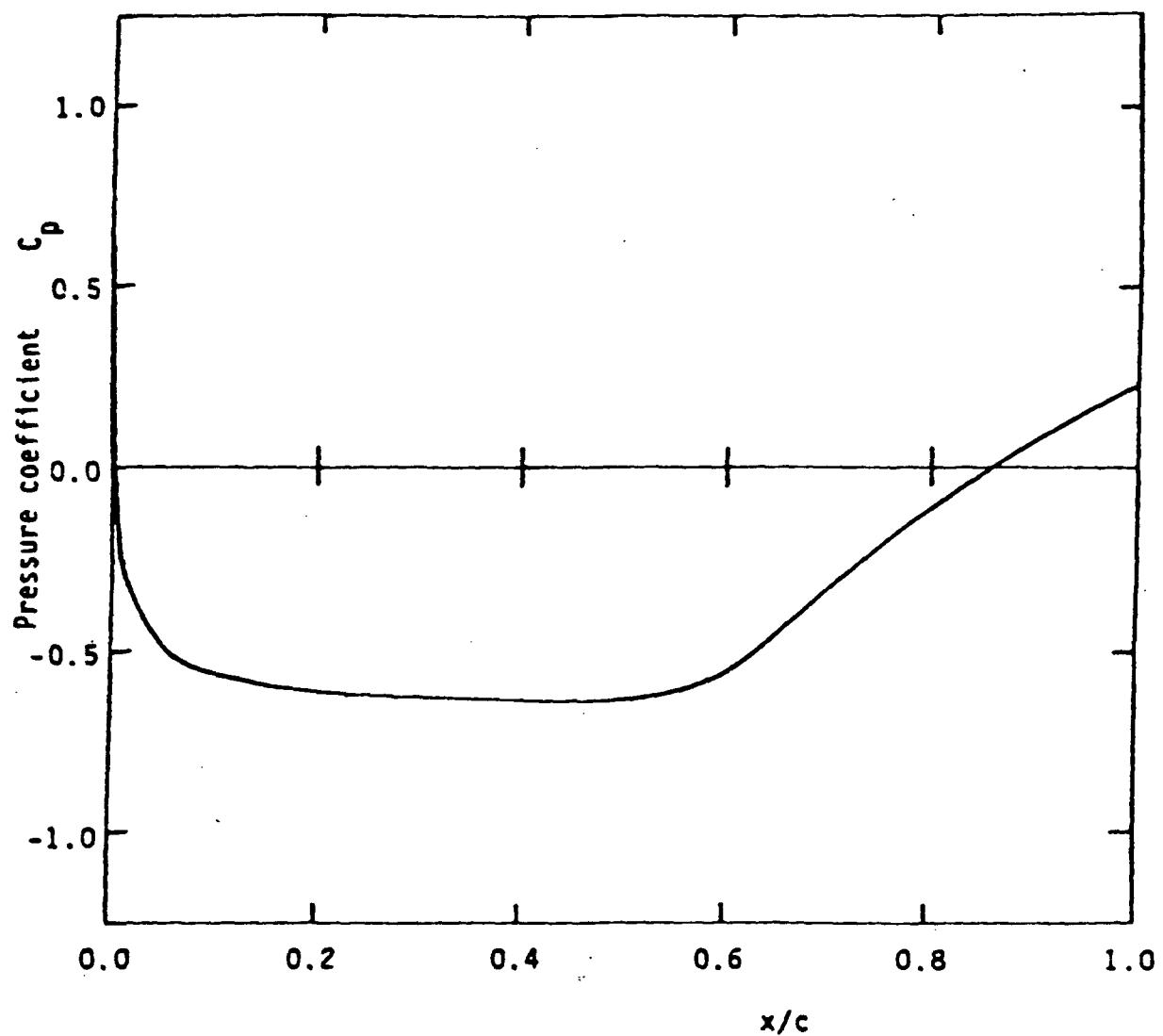


Fig. 2. I-21 upper surface pressure distribution.

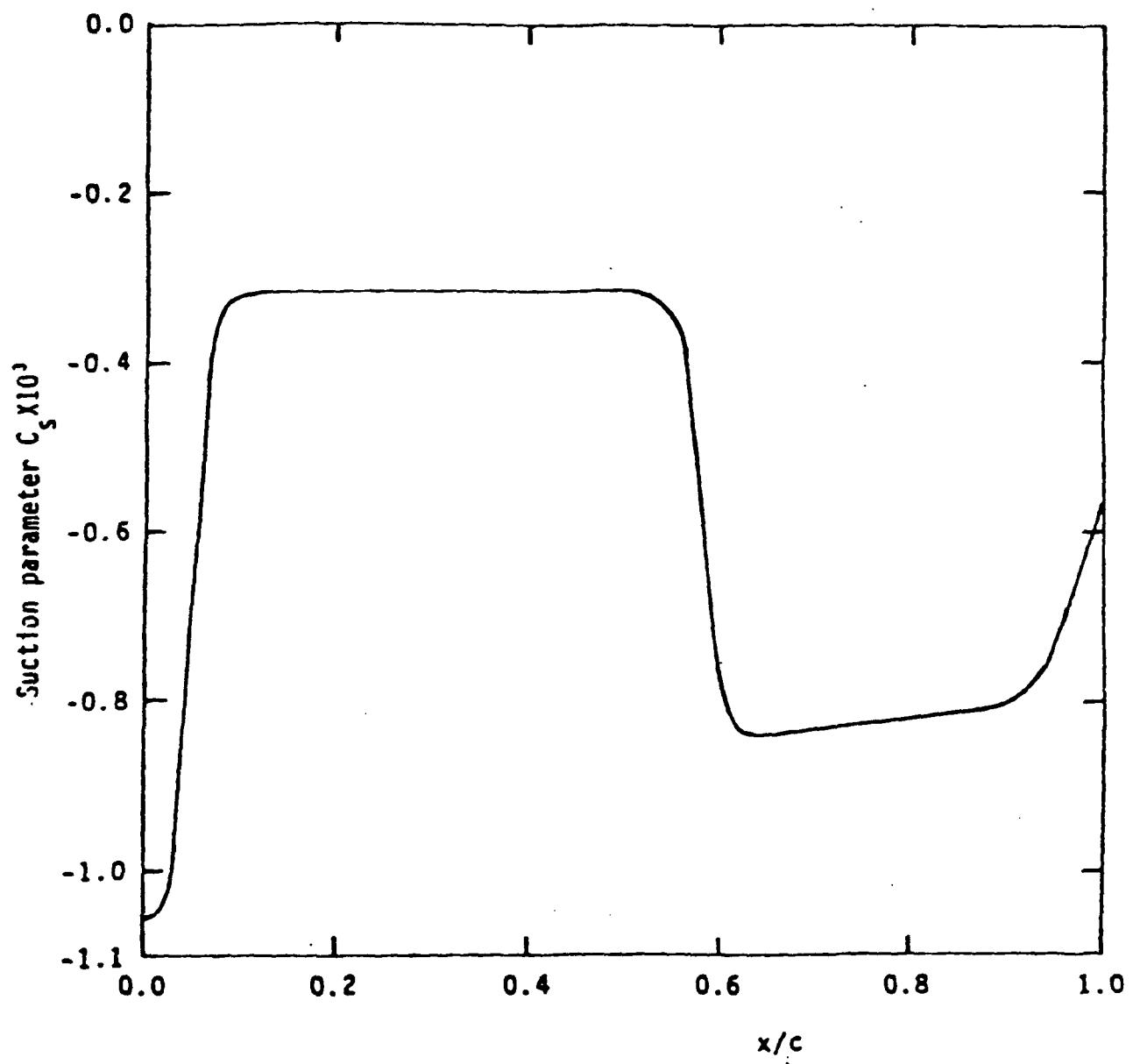


Fig. 3. X-21 upper surface suction distribution.

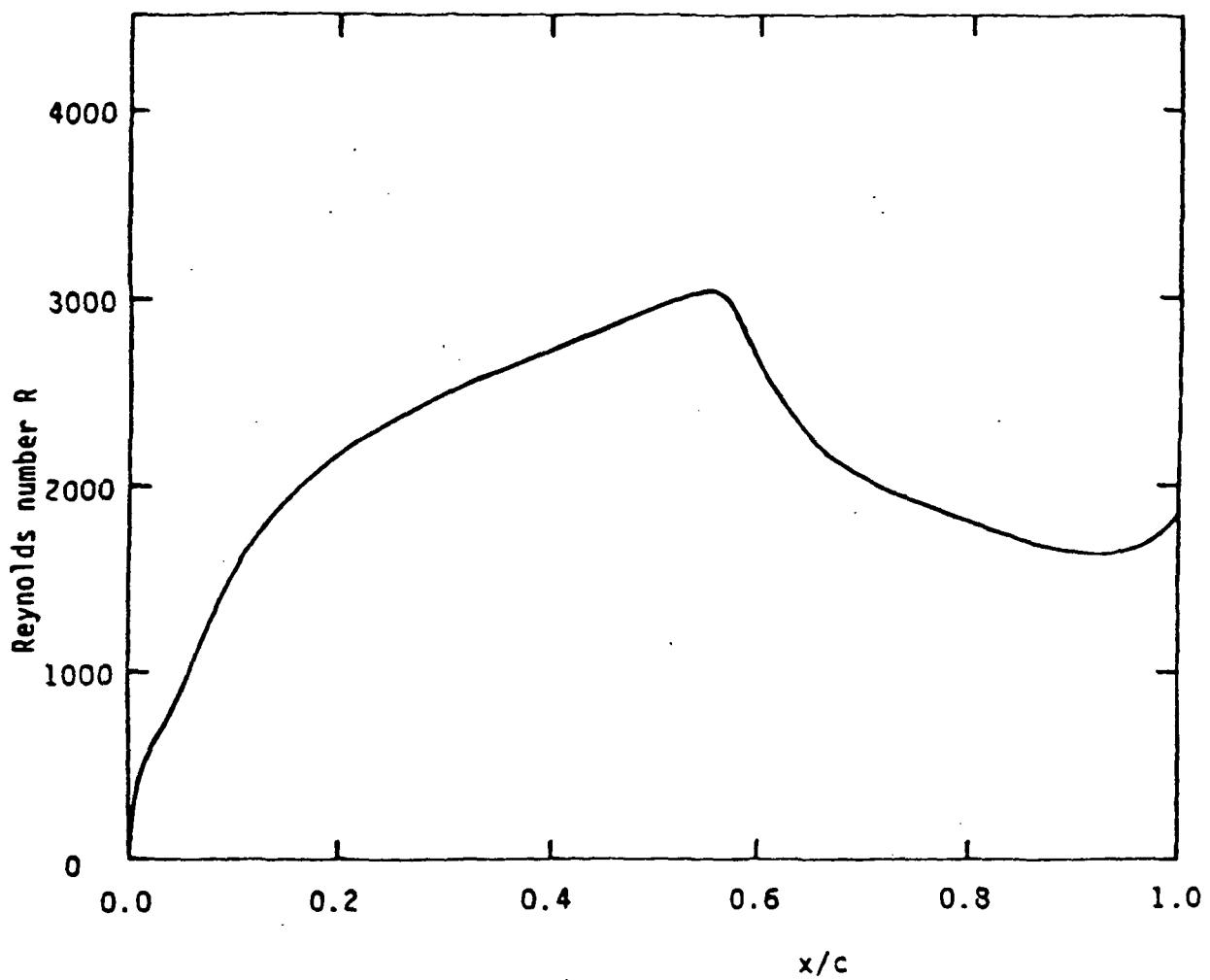
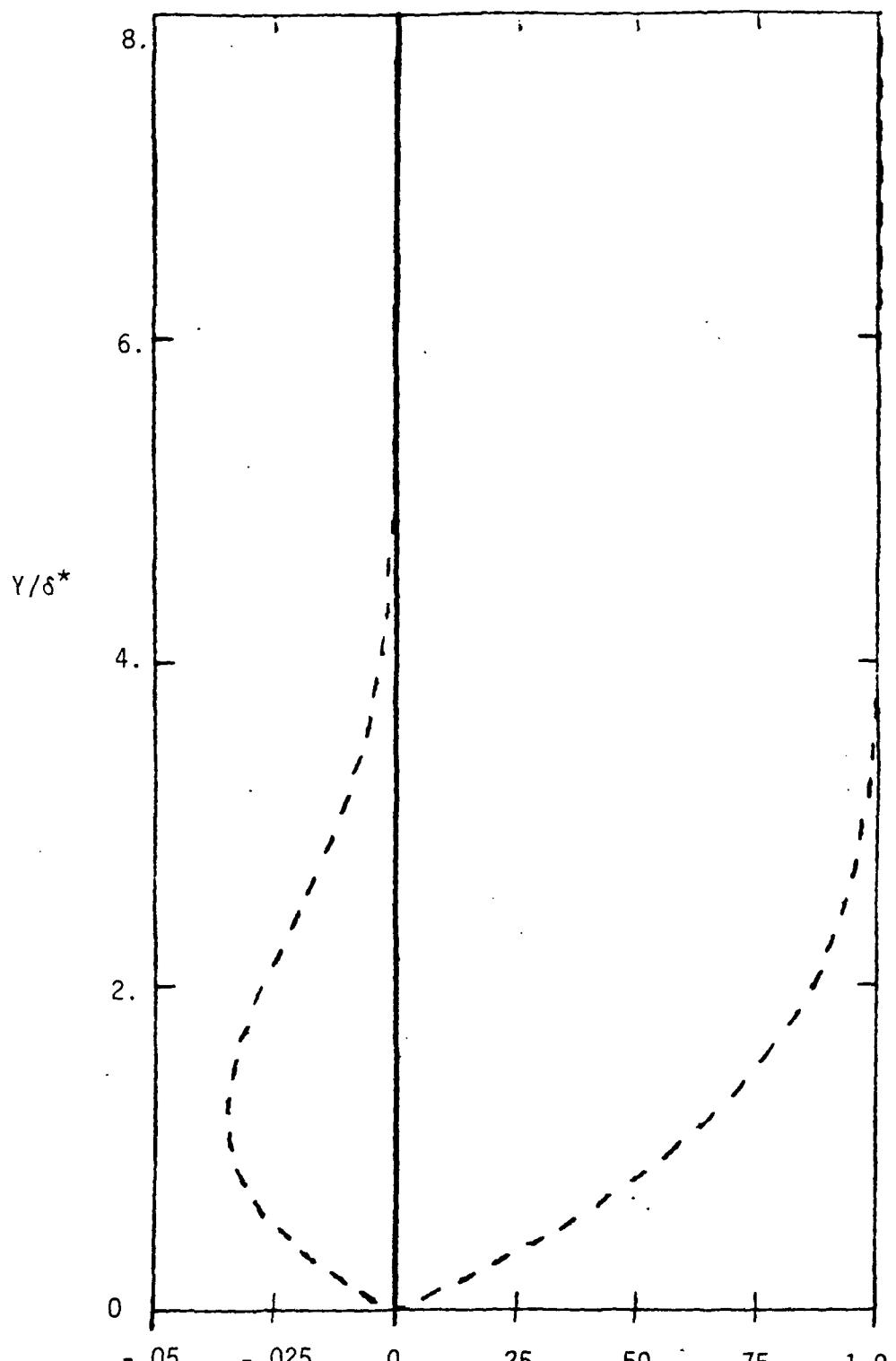


Fig. 4. X-21 upper surface displacement thickness

Reynolds Number



$$(-U_e^* U_s^* + U_e^* W_s^*) / \sqrt{U_e^{*2} + W_e^{*2}}$$

$$(U_e^* U_s^* + W_e^* W_s^*) / \sqrt{U_e^{*2} + W_e^{*2}}$$

Fig. 5. X-21 upper surface streamwise and crossflow velocity profiles at  $x/c = 0.015$ .

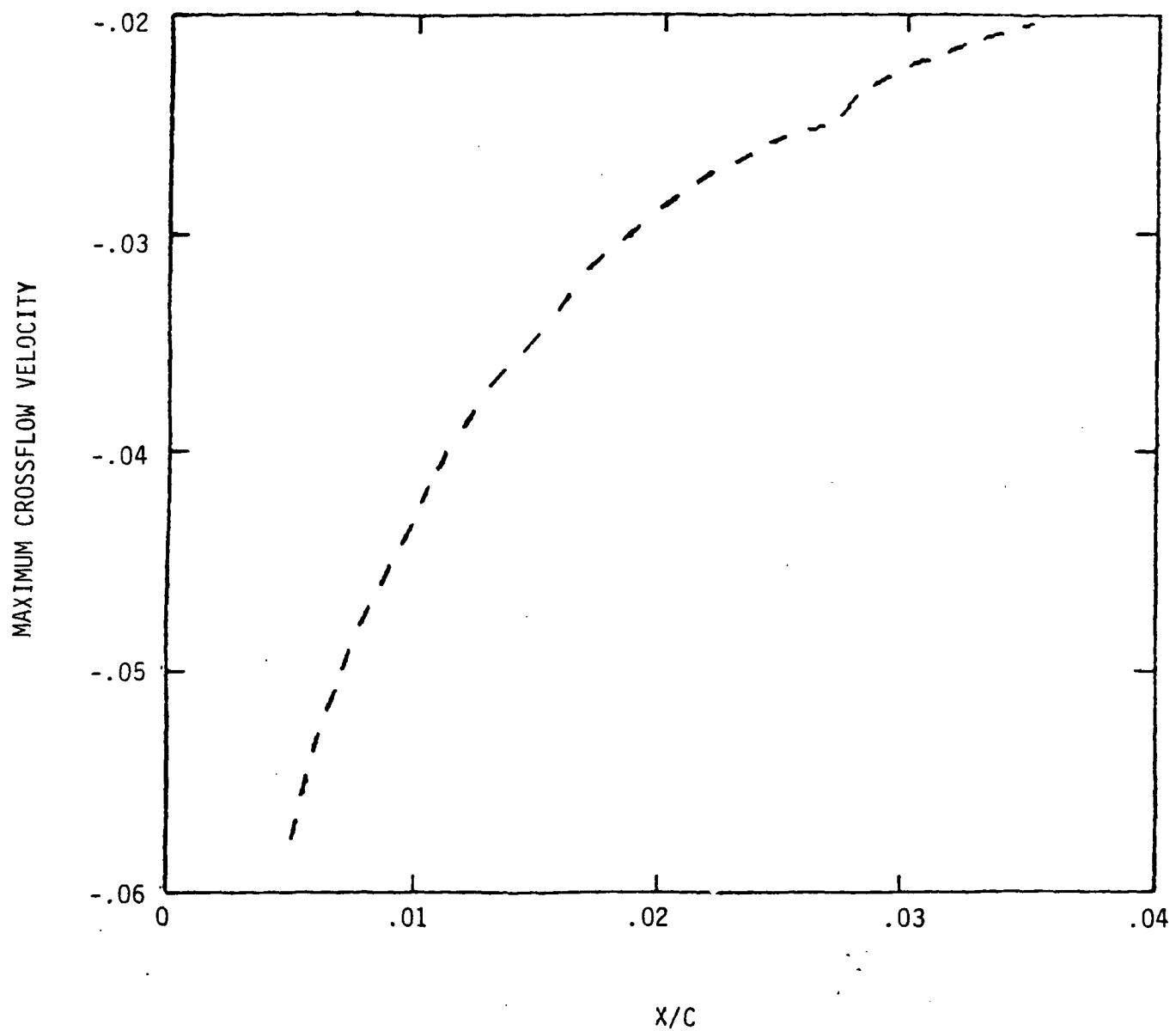


Fig. 6. X-21 upper surface maximum crossflow velocity distribution.

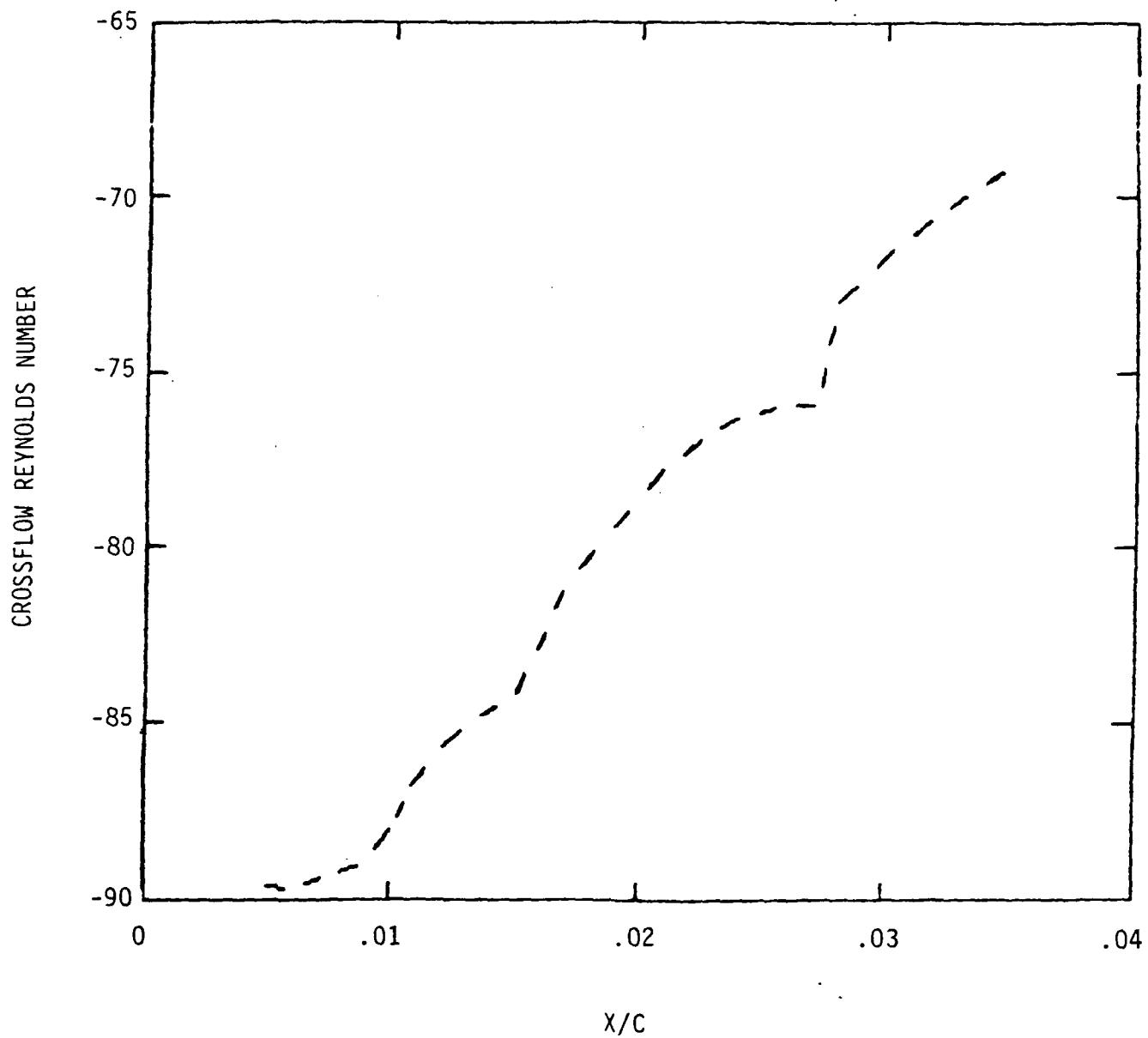
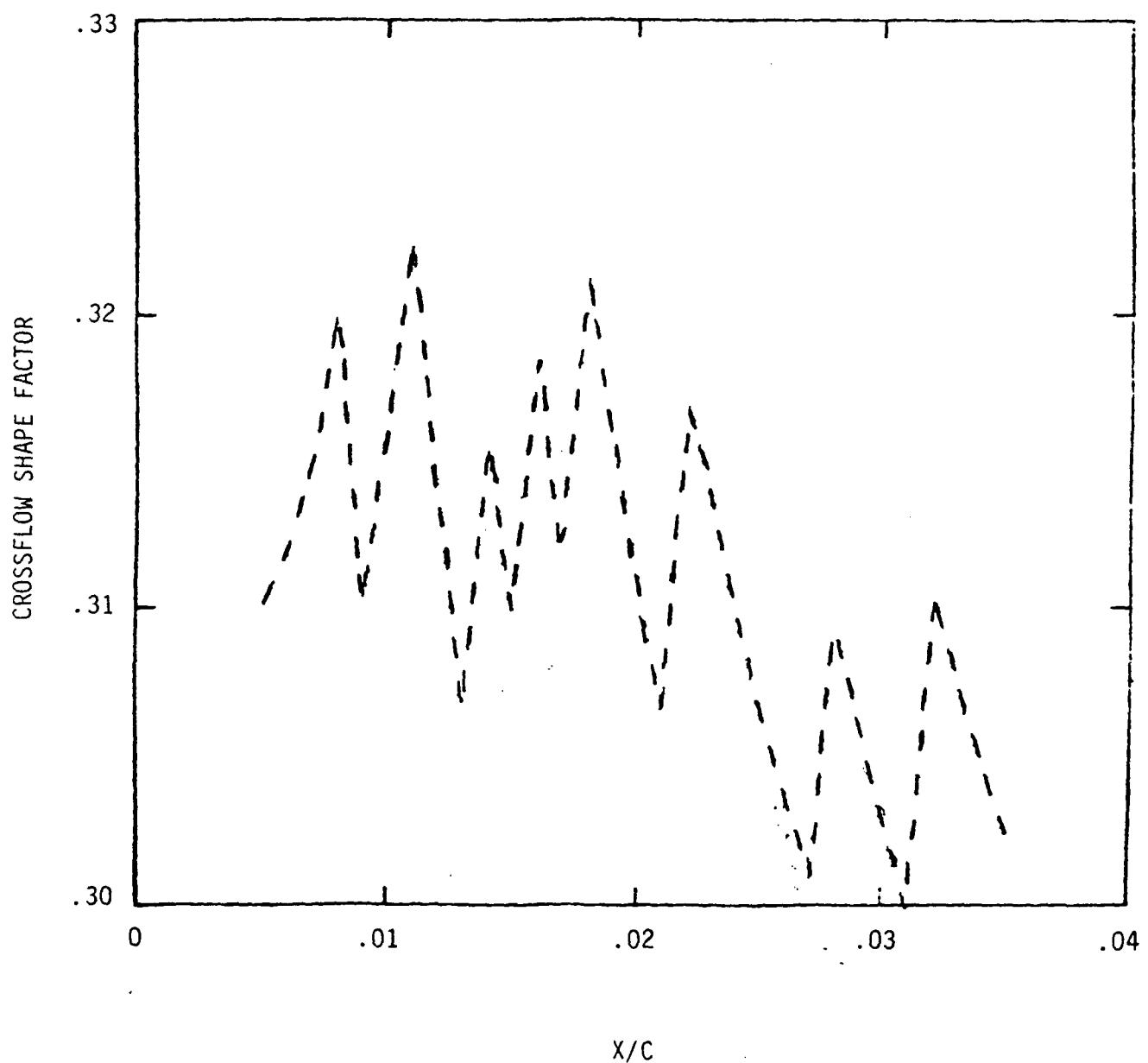


Fig. 7. X-21 upper surface crossflow Reynolds number distribution.



**Fig. 8. X-21 upper surface shape factor distribution.**

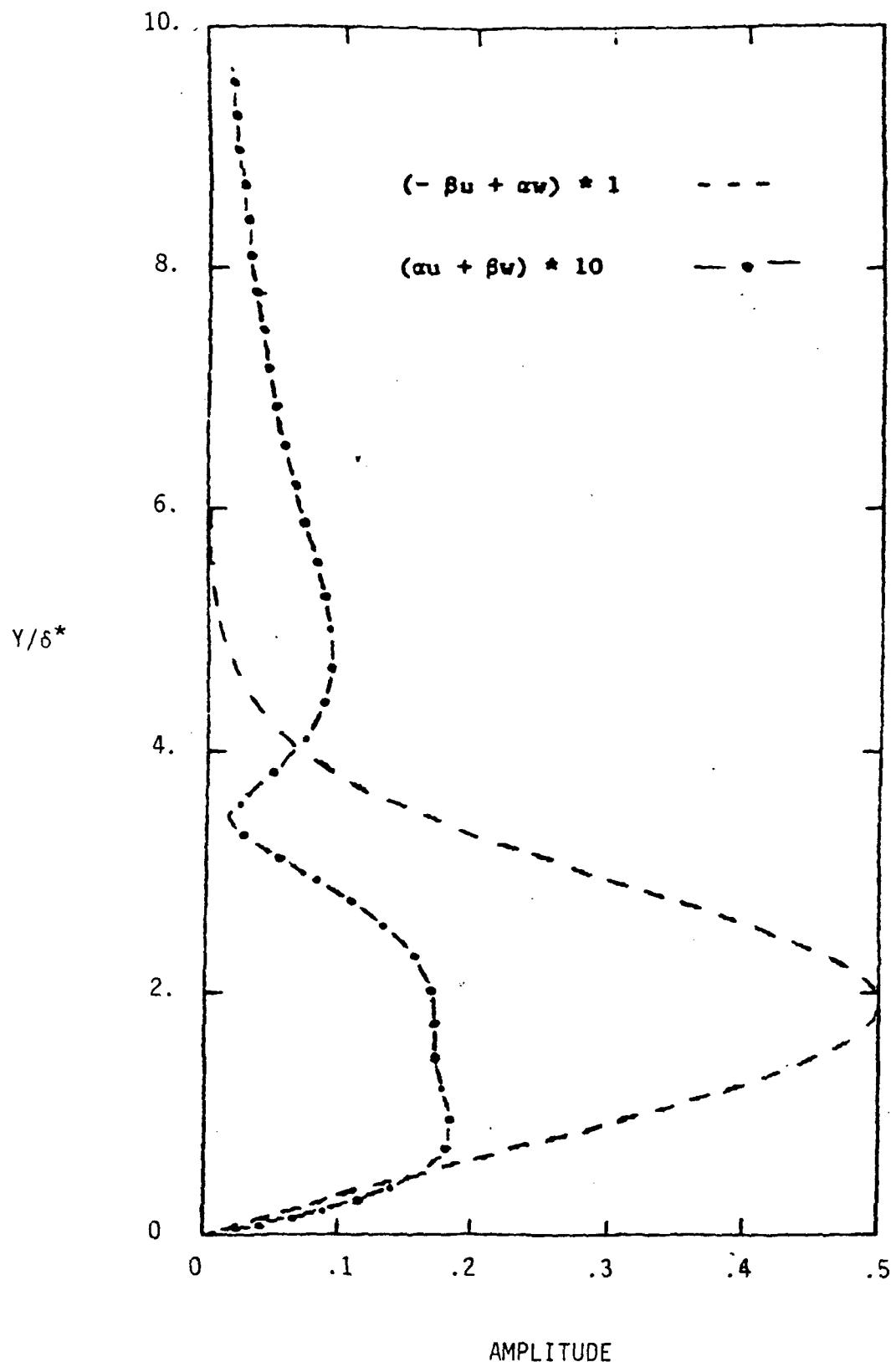


Fig. 9. Streamwise and crossflow disturbance profiles at  $x/c = 0.015$ .

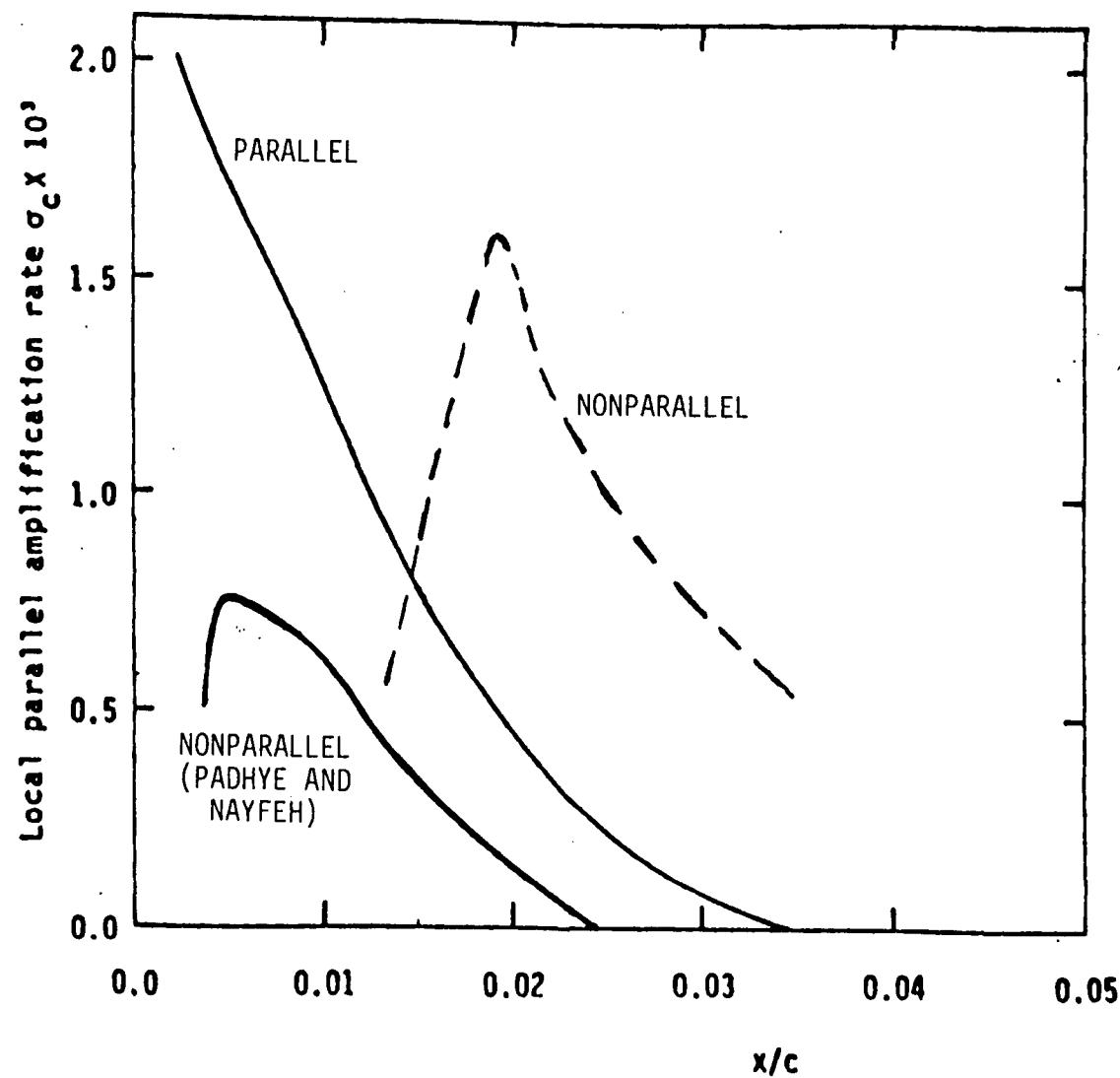


Fig. 10. Parallel and nonparallel growth rates for crossflow disturbance.

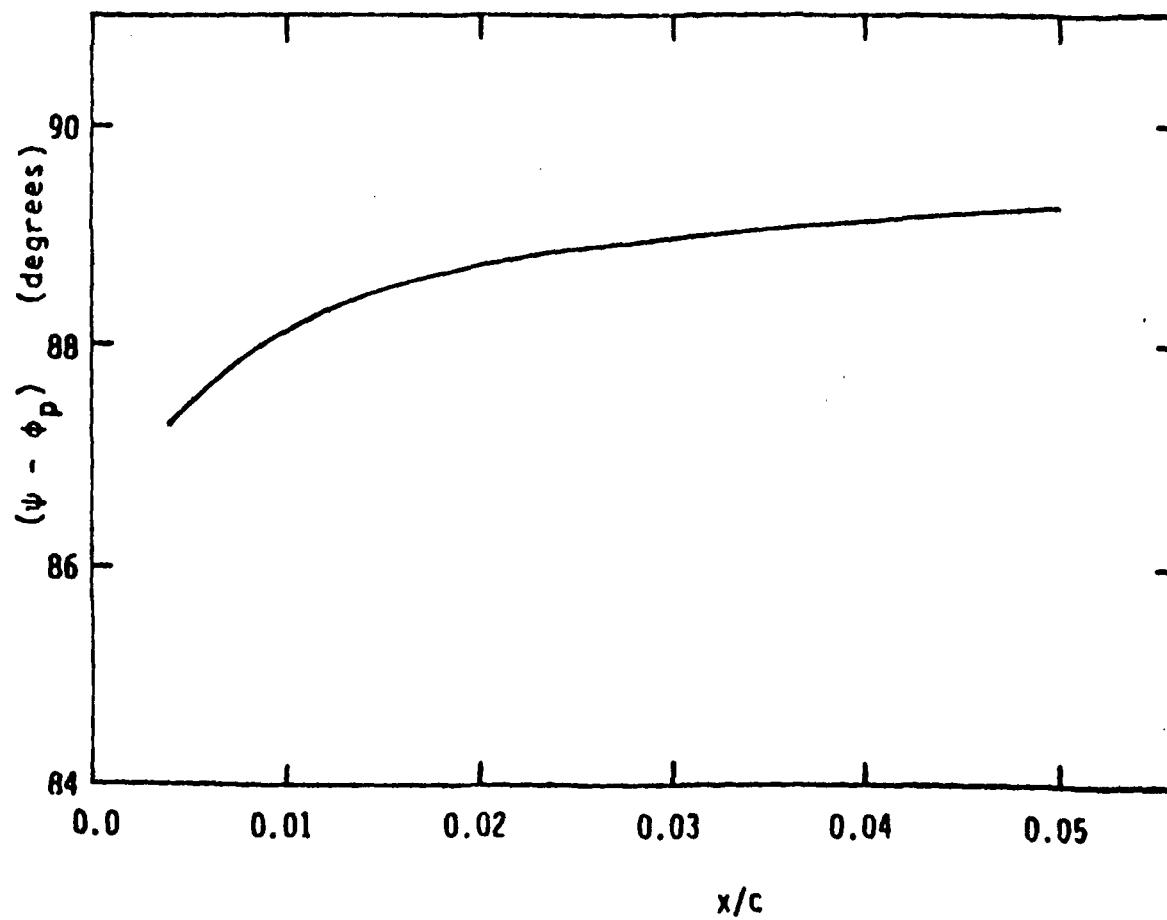


Fig. 11. Variation of difference in wave angle and potential flow direction  $(\psi - \phi_p)$ .

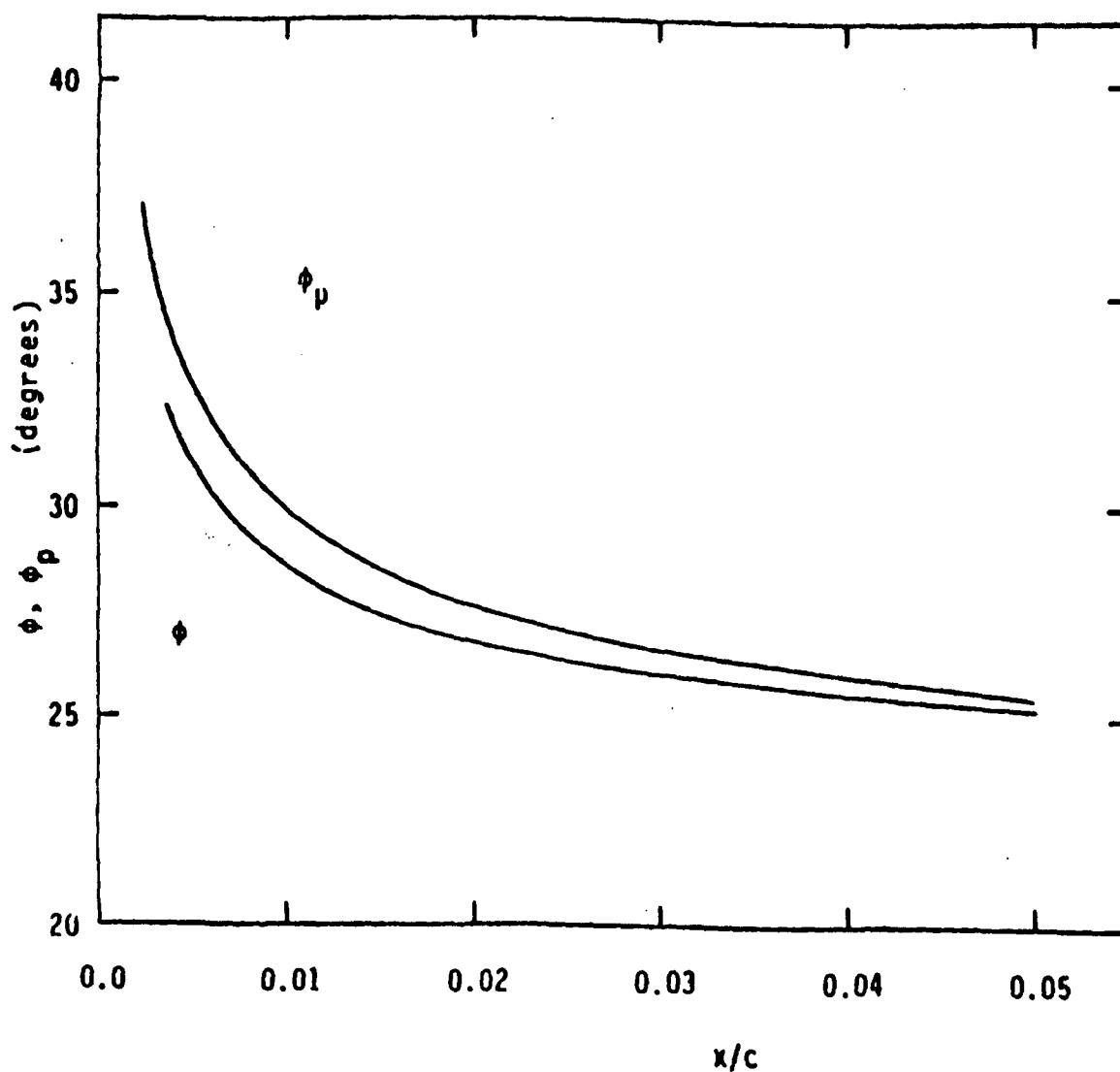


Fig. 12. Variation of group direction  $\phi$  and potential flow direction  $\phi_p$ .

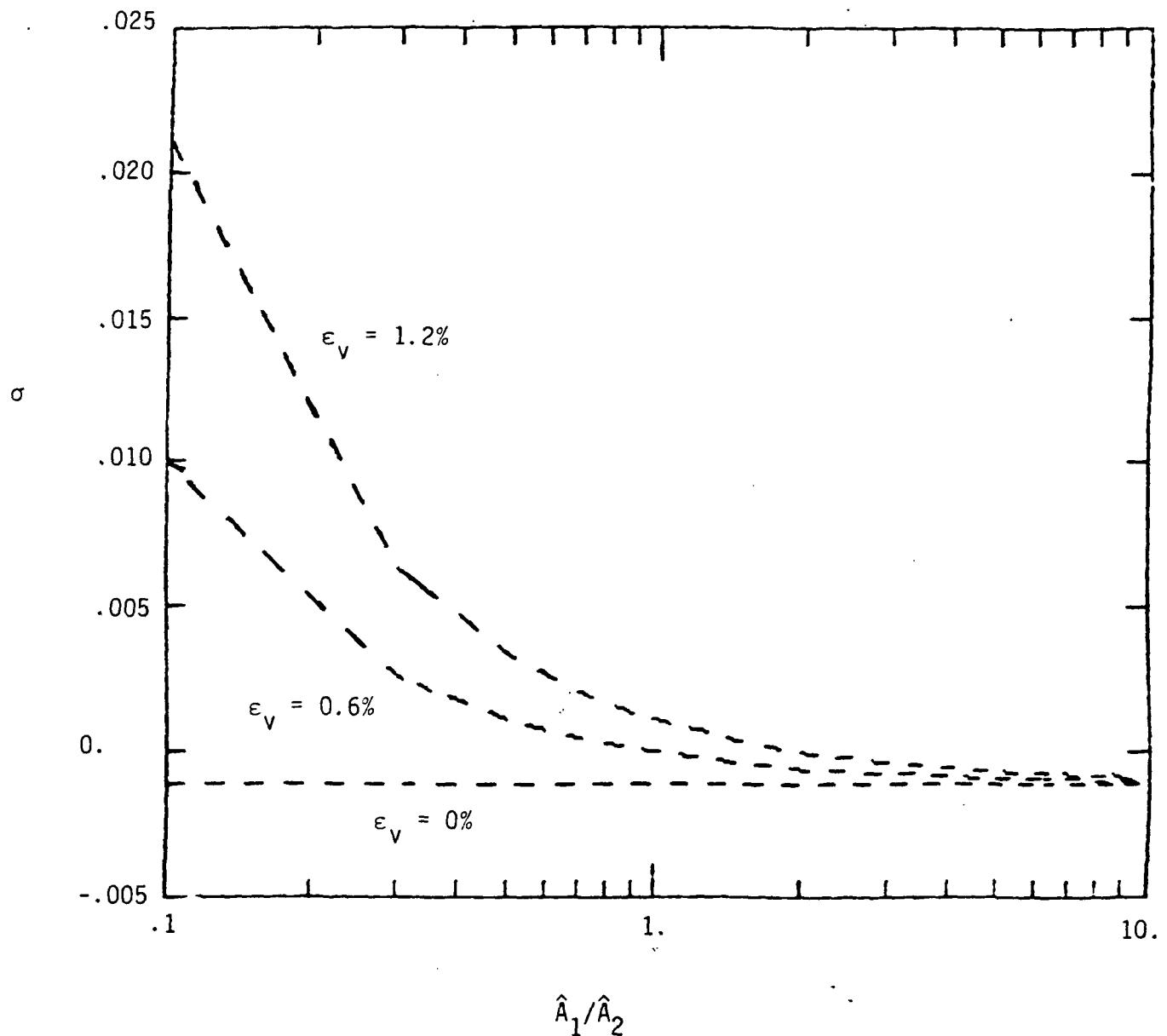


Fig. 13. Dependence of growth rate on T-S wave amplitude ratio  $A_1/A_2$  and crossflow amplitude  $\epsilon_V$ .

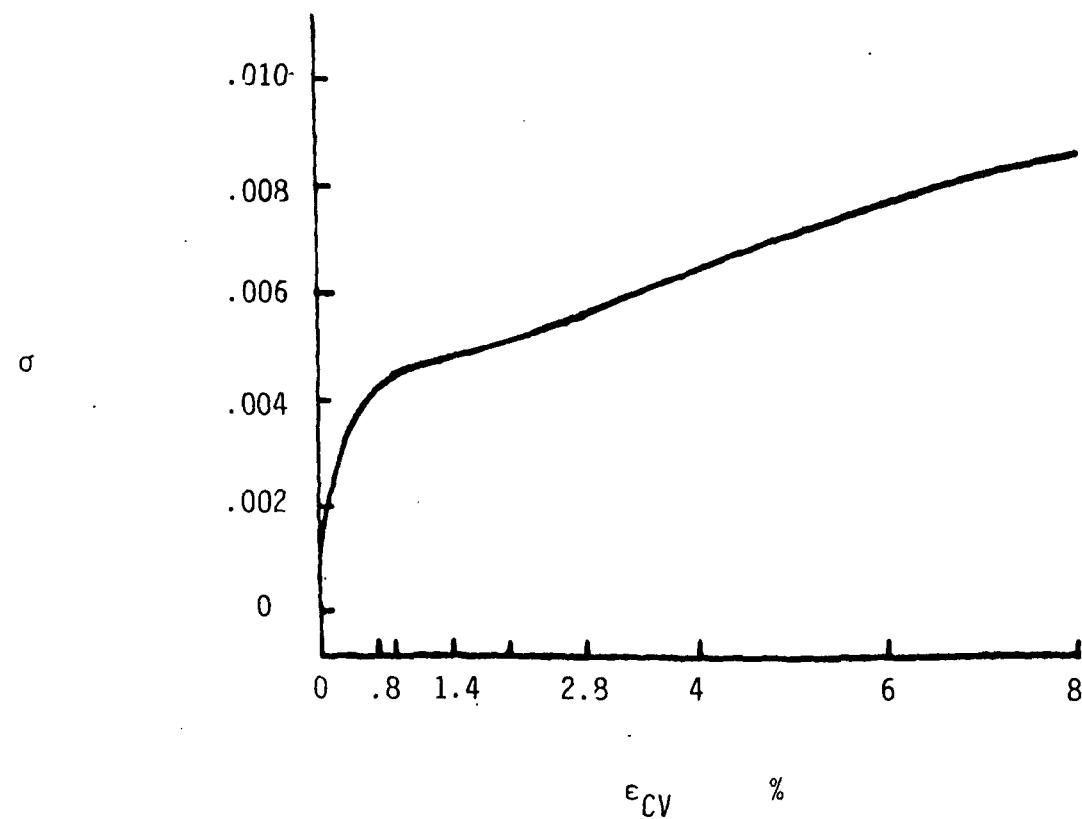


Fig. 14. Variation of growth rate with crossflow amplitude  $\epsilon_{cv}$  for 325 Hz and  $x/c = 0.015$ .

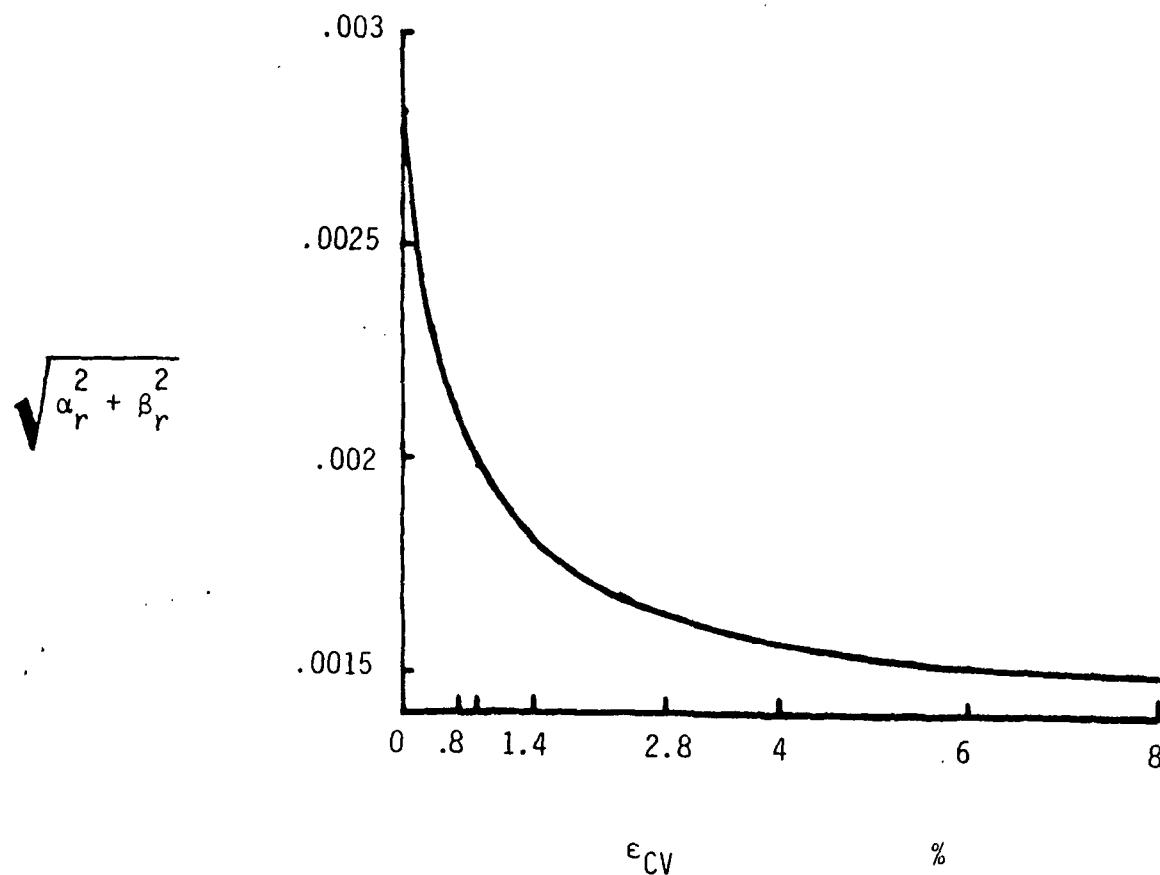


Fig. 15. Variation of dimensionless wave number with crossflow amplitude  $\epsilon_{CV}$  for 325 Hz and  $x/c = 0.015$ .

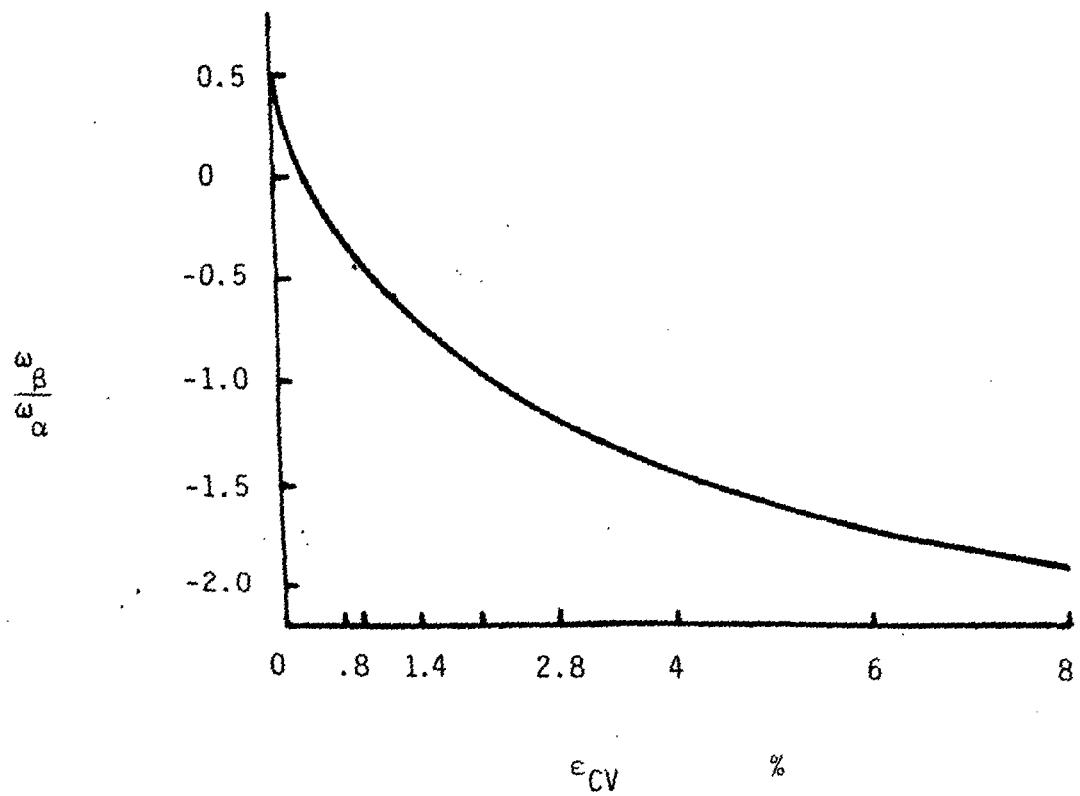


Fig. 16. Variation of group velocity ratio with crossflow amplitude  $\epsilon_{CV}$  for 325 Hz and  $x/c = 0.015$ .

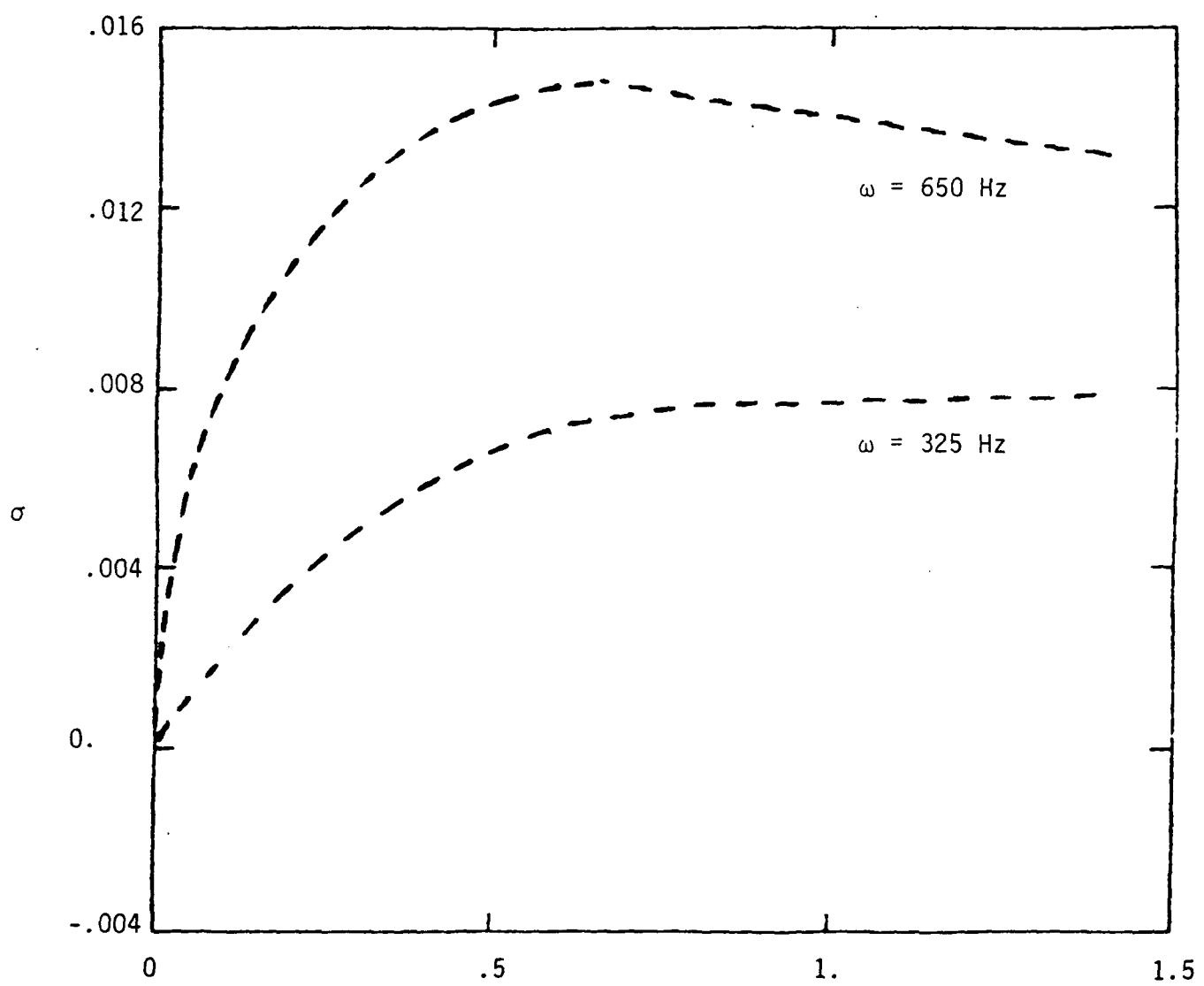


Fig. 17. Variation of growth rate with crossflow amplitude ratio and frequency for  $x/c = 0.025$ .

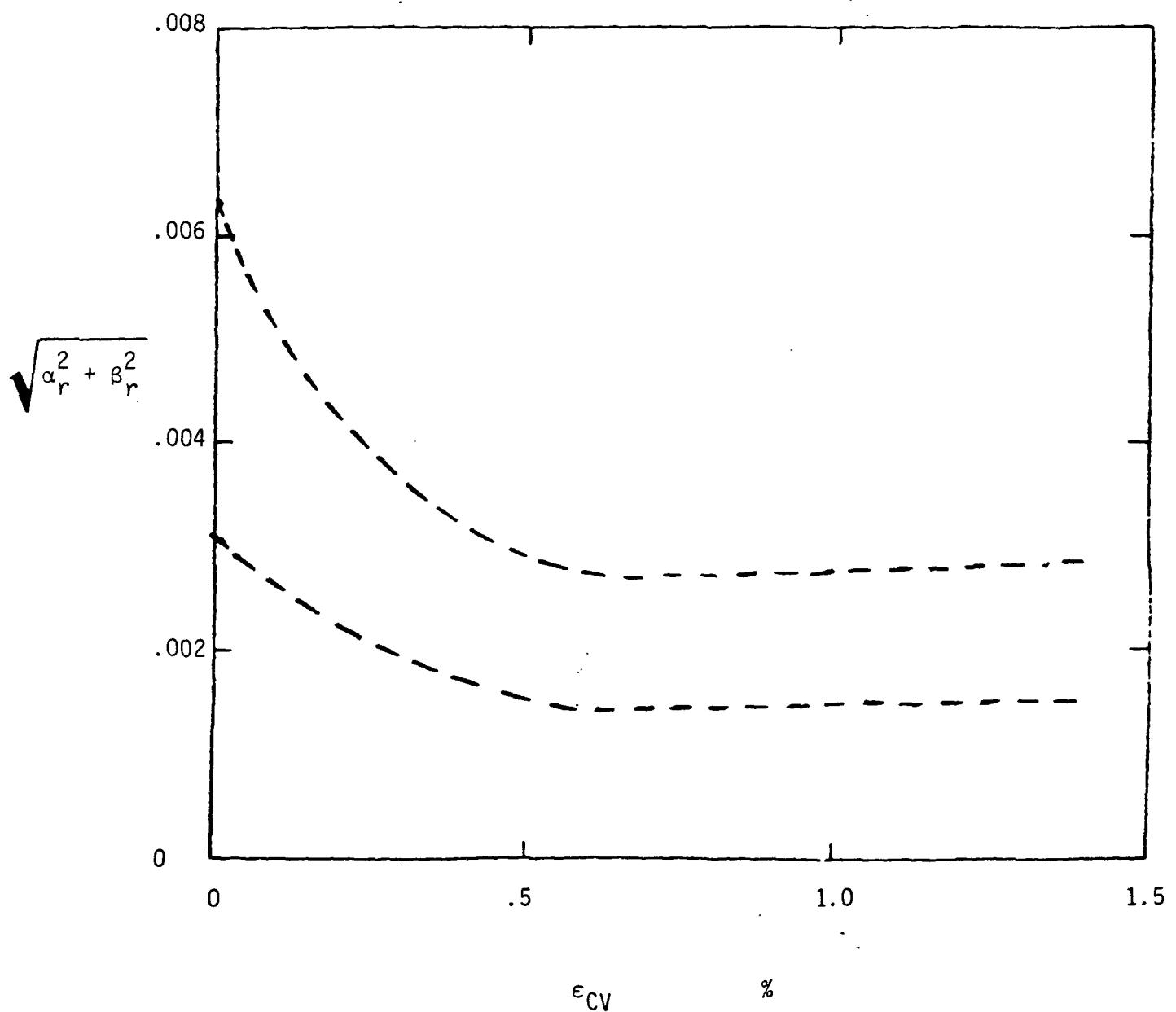


Fig. 18. Variation of dimensionless wave number with crossflow amplitude ratio and frequency for  $x/c = 0.025$ .

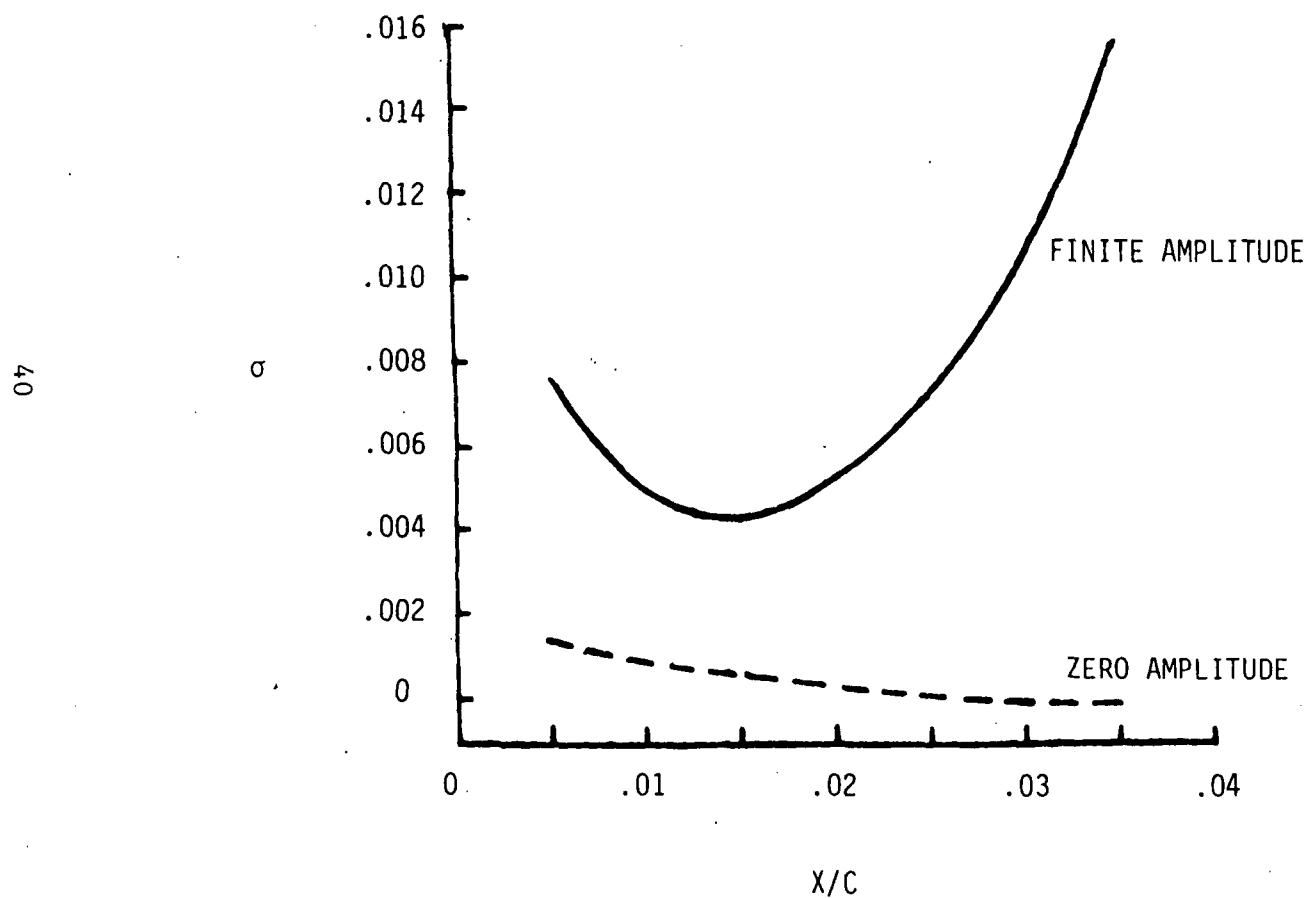


Fig. 19. Comparison of parallel zero-amplitude and finite-amplitude growth rates for 325 Hz and initial crossflow amplitude 0.6%.

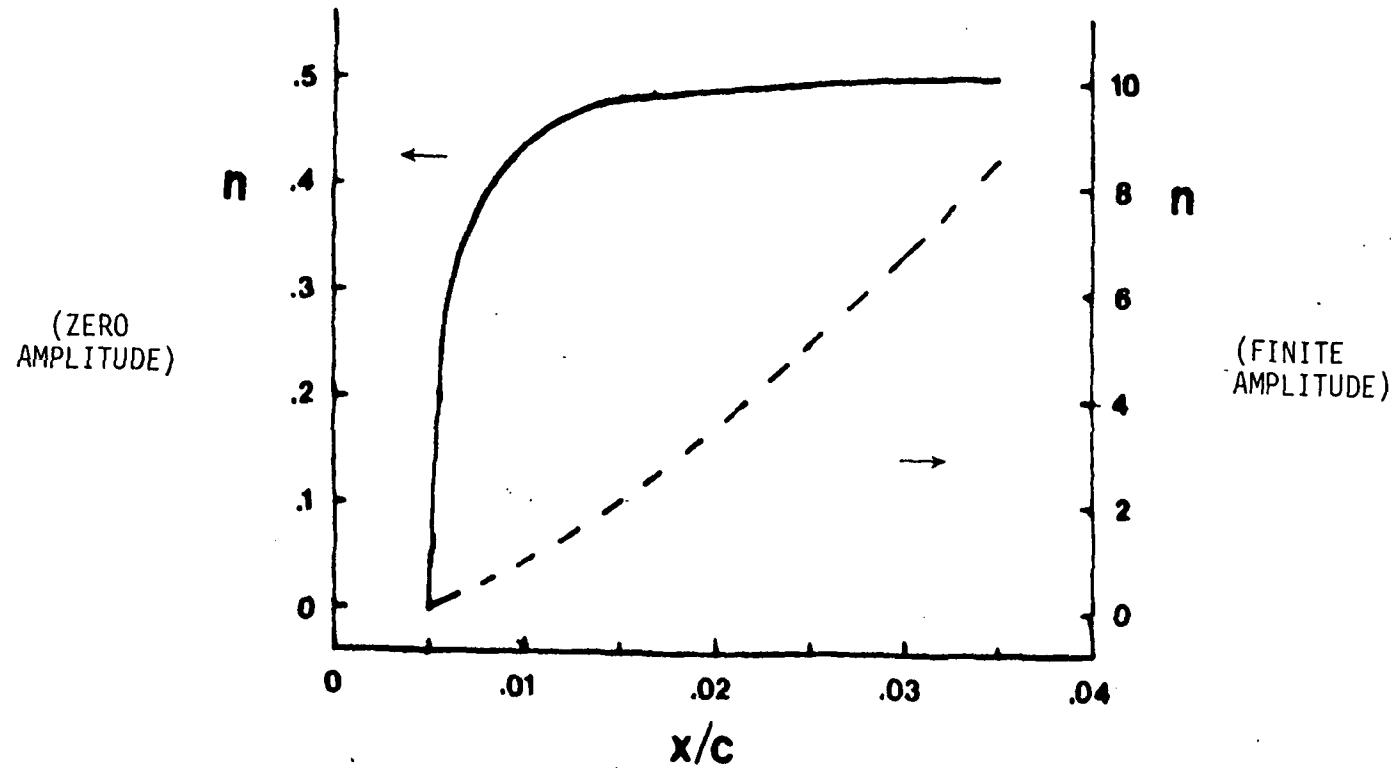


Fig. 20. Comparison of parallel zero-amplitude and finite-amplitude amplification factors for 325 Hz and initial crossflow amplitude 0.6%.

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## Appendix A

### ZEROTH-ORDER PROBLEM

$$a_{12} = 1$$

$$a_{21} = -iR(\omega - \alpha_v U_s - \beta_v W_s) + (\alpha_v^2 + \beta_v^2)$$

$$a_{23} = RDU_s$$

$$a_{24} = i\alpha_v R$$

$$a_{31} = -i\alpha_v$$

$$a_{35} = -i\beta_v$$

$$a_{42} = -\frac{i\alpha_v}{R}$$

$$a_{43} = i(\omega - \alpha_v U_s - \beta_v W_s) - (\alpha_v^2 + \beta_v^2)/R$$

$$a_{46} = -i\beta_v/R$$

$$a_{56} = 1$$

$$a_{63} = RDW_s$$

$$a_{64} = i\beta_v R$$

$$a_{65} = -iR(\omega - \alpha_v U_s - \beta_v W_s) + (\alpha_v^2 + \beta_v^2)$$

NOTE:  $\omega = 0$  for crossflow.

$\alpha_v$ ,  $\beta_v$  are the chordwise and spanwise components of the wavenumber.

### Appendix B

QUADRATURES FOR  $g_{ij}$ ,  $h_i$ ,  $h_{12}$ ,  $h_{21}$ ,  $i = 1, 2$ ,  $j = 1, 2, 3$

Define  $\zeta_{in}$  as follows for the two T-S waves:

$$\zeta_{i1} = \hat{u}_{io}, \quad \zeta_{i2} = \hat{v}_{io}, \quad \zeta_{i3} = \hat{w}_{io}, \quad \zeta_{i4} = \hat{p}_{io}$$

Then  $\bar{\zeta}_{in}$  are defined by:

$$\bar{\zeta}_{i1} = \zeta_{i1} + \frac{\beta_i}{\alpha_i} \zeta_{i3}$$

$$\bar{\zeta}_{i2} = \zeta_{i2}$$

$$\bar{\zeta}_{i3} = \zeta_{i3} - \frac{\beta_i}{\alpha_i} \zeta_{i1}$$

$$\bar{\zeta}_{i4} = \zeta_{i4}$$

and  $\bar{\zeta}_{in}^*$  are the corresponding adjoint solutions. Then

$$g_{i1} = \int_0^\infty \sum_{n=1}^4 D_{in} \bar{\zeta}_{in}^* dy$$

$$g_{i2} = \int_0^\infty \sum_{n=1}^4 E_{in} \bar{\zeta}_{in}^* dy$$

$$g_{i3} = \int_0^\infty \sum_{n=1}^4 F_{in} \bar{\zeta}_{in}^* dy$$

$$(\omega_\alpha)_i = g_{i2}/g_{i1}, \quad (\omega_\beta)_i = g_{i3}/g_{i1}$$

$$h_i = \int_0^\infty \sum_{n=1}^4 G_{in} \bar{\zeta}_{in}^* dy$$

$$h_{ij} = \int_0^\infty \sum_{n=1}^4 H_{in} \bar{\zeta}_{in}^* dy$$

$$i = 1, 2, \quad j = 1, 2, \quad i \neq j$$

$D_{in}$ :

$$D_{i1} = 0$$

$$D_{i2} = R\bar{\zeta}_{i1}$$

$$D_{i3} = 0$$

$$D_{i4} = -\bar{\zeta}_{i2}$$

$E_{in}$ :

$$E_{i1} = 0$$

$$E_{i2} = (U_s R - 2i\alpha_i) \bar{\zeta}_{i1} + R\bar{\zeta}_{i4}$$

$$E_{i3} = -\zeta_{i1}$$

$$E_{i4} = -\frac{1}{R} D\zeta_{i1} - U_s \zeta_{i2} + \frac{2i\alpha_i}{R} \zeta_{i2}$$

$F_{in}$ :

$$F_{i1} = 0$$

$$F_{i2} = W_s R \bar{\zeta}_{i1} - 2i\beta_i \bar{\zeta}_{i1} + R \frac{\beta_i}{\alpha_i} \bar{\zeta}_{i4}$$

$$F_{i3} = -\zeta_{i3}$$

$$F_{i4} = -\frac{1}{R} D\zeta_{i3} - W_s \zeta_{i2} + \frac{2i\beta_i}{R} \zeta_{i2}$$

$G_{1n}$ :

$$G_{11} = 0$$

$$\begin{aligned}
 G_{12} = & U_s \frac{\partial \zeta_{11}}{\partial x_1} R + W_s \frac{\partial \zeta_{11}}{\partial z_1} R + \frac{\partial \zeta_{14}}{\partial x_1} R - 2i\alpha_i \frac{\partial \zeta_{11}}{\partial x_1} \\
 & - i\zeta_{11} \frac{\partial \alpha_i}{\partial x_1} - 2i\beta_i \frac{\partial \zeta_{11}}{\partial z_1} - i\zeta_{11} \frac{\partial \beta_i}{\partial z_1} \\
 & + U_s \frac{\partial \zeta_{13}}{\partial x_1} R \frac{\beta_i}{\alpha_i} + W_s \frac{\partial \zeta_{13}}{\partial z_1} R \frac{\beta_i}{\alpha_i} + \frac{\partial \zeta_{14}}{\partial z_1} R \frac{\beta_i}{\alpha_i} \\
 & - 2i\beta_i \frac{\partial \zeta_{13}}{\partial x_1} - i \frac{\beta_i}{\alpha_i} \zeta_{13} \frac{\partial \alpha_i}{\partial x_1} - 2i \frac{\beta_i}{\alpha_i} \frac{\partial \zeta_{13}}{\partial z_1} \\
 & - i \frac{\beta_i}{\alpha_i} \zeta_{13} \frac{\partial \beta_i}{\partial z_1} + R\zeta_{11} \frac{\partial U_s}{\partial x_1} + V_s^* R \frac{\partial \zeta_{11}}{\partial y} + R\zeta_{13} \frac{\partial U_s}{\partial z_1} + R\zeta_{11} \frac{\beta_i}{\alpha_i} \frac{\partial W_s}{\partial x_1} \\
 & + V_s^* R \frac{\beta_i}{\alpha_i} \frac{\partial \zeta_{13}}{\partial y} + R\zeta_{13} \frac{\beta_i}{\alpha_i} \frac{\partial W_s}{\partial z_1}
 \end{aligned}$$

$$G_{13} = - \frac{\partial \zeta_{11}}{\partial x_1} - \frac{\partial \zeta_{13}}{\partial z_1}$$

$$\begin{aligned}
 G_{14} = & - \frac{1}{R} \frac{\partial D\zeta_{11}}{\partial x_1} - \frac{1}{R} \frac{\partial D\zeta_{13}}{\partial z_1} - U_s \frac{\partial \zeta_{12}}{\partial x_1} - W_s \frac{\partial \zeta_{12}}{\partial z_1} \\
 & + \frac{2i\alpha_i}{R} \frac{\partial \zeta_{12}}{\partial x_1} + i \frac{1}{R} \zeta_{12} \frac{\partial \alpha_i}{\partial x_1} + \frac{2i\beta_i}{R} \frac{\partial \zeta_{12}}{\partial z_1} + i \frac{1}{R} \zeta_{12} \frac{\partial \beta_i}{\partial z_1} \\
 & - V_s^* \frac{\partial \zeta_{12}}{\partial y} - \zeta_{12} \frac{\partial V_s^*}{\partial y}
 \end{aligned}$$

$H_{1n}$ :

$$H_{11} = 0$$

$$\begin{aligned}
 H_{12} = & KR \left( i\alpha_2 \bar{u}_o \zeta_{21} - i\bar{\alpha}_V \zeta_{21} \bar{u}_o + \bar{v}_o \frac{\partial \zeta_{21}}{\partial y} \right. \\
 & \left. + \zeta_{22} \frac{\partial \bar{u}_o}{\partial y} + i\beta_2 \bar{w}_o \zeta_{21} - i\bar{\beta}_V \zeta_{23} \bar{u}_o \right) \\
 & KR \frac{\beta_1}{\alpha_1} \left( i\alpha_2 \bar{u}_o \zeta_{23} - i\bar{\alpha}_V \zeta_{21} \bar{w}_o + \bar{v}_o \frac{\partial \zeta_{23}}{\partial y} \right. \\
 & \left. + \zeta_{22} \frac{\partial \bar{w}_o}{\partial y} + i\beta_2 \bar{w}_o \zeta_{23} - i\bar{\beta}_V \zeta_{23} \bar{w}_o \right)
 \end{aligned}$$

$$H_{13} = 0$$

$$H_{14} = -K \left( i\alpha_2 \bar{u}_o \zeta_{22} - i\bar{\alpha}_v \zeta_{21} \bar{v}_o + \bar{v}_o \frac{\partial \zeta_{22}}{\partial y} + \zeta_{22} \frac{\partial \bar{v}_o}{\partial y} + i\beta_2 \bar{w}_o \zeta_{22} - i\bar{\beta}_v \zeta_{23} \bar{v}_o \right)$$

$H_{2n}:$

$$H_{21} = 0$$

$$H_{22} = \frac{1}{K} R \left( i\alpha_1 u_o \zeta_{11} + i\alpha_v \zeta_{11} u_o + v_o \frac{\partial \zeta_{11}}{\partial y} + \zeta_{12} \frac{\partial u_o}{\partial y} + i\beta_1 w_o \zeta_{11} + i\beta_v \zeta_{13} u_o \right) + \frac{1}{K} R \frac{\beta_2}{\alpha_2} \left( i\alpha_1 u_o \zeta_{13} + i\alpha_v \zeta_{11} w_o + v_o \frac{\partial \zeta_{13}}{\partial y} + \zeta_{12} \frac{\partial w_o}{\partial y} + i\beta_1 w_o \zeta_{13} + i\beta_v \zeta_{13} w_o \right)$$

$$H_{23} = 0$$

$$H_{24} = -\frac{1}{K} \left( i\alpha_1 u_o \zeta_{12} + i\alpha_v \zeta_{11} v_o + v_o \frac{\partial \zeta_{12}}{\partial y} + \zeta_{12} \frac{\partial v_o}{\partial y} + i\beta_1 w_o \zeta_{12} + i\beta_v \zeta_{13} v_o \right)$$

Note:  $K = \frac{\text{Amplitude T-S wave #2}}{\text{Amplitude T-S wave #1}}$  .

Appendix C  
RESUME OF H. L. REED

HELEN LOUISE REED

Assistant Professor, Mechanical Engineering  
Stanford University  
Stanford, California 94305  
415-497-3313

A. PERSONAL DATA:

Date of Birth: [REDACTED]  
Social Security Number: [REDACTED]  
Home: 42 Peter Coutts Circle, Stanford, CA 94305.

B. EDUCATION:

A.B. in Mathematics, Goucher College, May 1977.  
Graduated with general honors and honors in mathematics, GPA 3.98/4.0.  
Graduated in three years via special honors program.  
Thesis: Limit Cycles of Autonomous Systems of Differential Equations.  
Advisor: Dr. Dorothy Bernstein.

M.S. in Engineering Mechanics, VPI & SU, June 1980.  
GPA 4.0/4.0.  
Thesis: Closed-Form Solutions for Subsonic Flows over Plates with  
Porous Strips.  
Advisor: Dr. Ali Nayfeh.

Ph.D. in Engineering Mechanics, VPI & SU, December 1981.  
GPA 4.0/4.0.  
Thesis: The Tollmien-Schlichting Instability of Laminar Viscous Flows.  
Advisor: Dr. Ali Nayfeh.

C. EXPERIENCE:

September 1982-present, Assistant Professor, Stanford University.  
Department - Mechanical Engineering.  
Duties - teach gas dynamics, undergraduate fluid mechanics, mathematics,  
hydrodynamic stability; research in boundary-layer separation,  
transition, and laminar flow control; student advisement.

January 1982-August 1982, Assistant Professor, VPI & SU.  
Department - Engineering Science and Mechanics.  
Duties - teach undergraduate fluid mechanics; research in boundary-layer  
transition and laminar flow control, specifically the use of suction  
in controlling the Tollmien-Schlichting instability; student advise-  
ment.

June 1979-December 1981, Aerospace Technologist (NASA-Langley) on graduate leave  
without pay to study for Master's and Ph.D. degrees in Engineering Mechan-  
ics at Virginia Polytechnic Institute and State University. Position (June  
1979-August 1981): Graduate Research Assistant; position (September 1981-  
December 1981): Graduate Research Associate.

September 1978-June 1979, Aerospace Technologist (NASA-Langley) on graduate leave with pay to study for Master's degree in Engineering Mechanics at Virginia Polytechnic Institute and State University.

June 1977-September 1978, Aerospace Technologist, NASA-Langley.  
Division - Aeronautical Systems Division.  
Chief - A. Nagel.  
Principal Supervisor - Dr. Werner Pfenninger, Chief Scientist.  
Duties - design of supercritical laminar flow control (LFC) airfoils;  
investigation of induced drag associated with a modular system of  
aircraft; supervise co-op students also assigned to these projects.

Summer 1976, Mathematics Aid, NASA-Langley.  
Division - Subsonic Transonic Aerodynamics Division.  
Branch - Dynamic Stability.  
Principal Supervisor - Claude Patterson.  
Duties - support experimental research investigating the effect of wing-tip  
vortices of large aircraft on small following aircraft and the  
attenuation of such vortices.

#### OTHER EXPERIENCE:

July 1983-August 1983, Summer University Faculty, Sandia National Laboratories,  
Albuquerque, NM  
Division - Applied Mathematics Division  
Supervisor - Dr. M. R. Scott

#### D. CONSULTING:

September 1981-December 1981, consultant to International Consultants in Science  
and Technology, Inc. Developed a computer code for Westinghouse Electric  
and Naval Underwater Sea Center, optimizing suction strip configuration on  
axisymmetric bodies using linear triple-deck theory; supervise two computer  
science students.

#### E. PROFESSIONAL ACTIVITIES

##### E.1 SCIENTIFIC AND PROFESSIONAL SOCIETIES:

Member, American Physical Society (APS)  
Member, American Institute of Aeronautics and Astronautics (AIAA)  
Member, Society for Industrial and Applied Mathematics (SIAM)  
Member, Sigma Xi  
Member, American Society for Engineering Education (ASEE)  
Member, International Association for Computational Mechanics (IACM)  
Associate Member, American Society of Mechanical Engineers (ASME)

E.2 SERVICE TO THE PROFESSION

1. Referee journal articles for Physics of Fluids and Annual Review of Fluid Mechanics
2. Originated idea of Fluid Mechanics Photo Contest at annual meetings of the American Physical Society, November, 1983, Houston. Responsibility for Fluid Mechanics Photo contest at American Physical Society meeting at Brown University, November, 1984.
3. Serve on AIAA Technical Committee on Fluid Dynamics, 1984-1987.
4. Chaired workshop on "Transition Prediction Models, Code Development, and Verification" at NASA/Lewis Symposium on Transition in Turbines, May 15-16, 1984.

E.3 HONORS AND RECOGNITIONS:

Phi Kappa Phi

Phi Beta Kappa

Merit Scholarship from the State of Maryland, 1974

Recipient of a NASA fellowship, 1976

Outstanding Summer Employee Award from NASA-Langley, 1976

Torrey Award for Excellence in Mathematics, Goucher College, 1977

Outstanding Achievement Award from NASA-Langley, 1978

Listed in Who's Who in American Colleges and Universities

Cunningham Fellowship Award from VPI & SU, 1981

Presidential Young Investigator Award, 1984

E.4 CURRENT FIELDS OF INTEREST:

Hydrodynamic Stability, Laminar Flow Control, Computational Fluid Mechanics, Transonic Aerodynamics, Perturbation Methods, Separated and Transitional Flows, Three-Dimensional Boundary Layers, Experimental Methods.

F. TEACHING

F.1 STANFORD UNIVERSITY

Ph.D. Students Supervised

B. Singer, "Numerical Simulation of Transition to Turbulence in a Channel," expected Spring, 1986.

R. Henk, "Fundamental Studies of Three-Dimensional Unsteady Separation at Low Reynolds Numbers," expected Spring, 1987.

K. Yang, "Numerical Simulation of the Effects of Freestream Disturbances and Roughness on Transition in Boundary Layers," expected Spring, 1987.

Ph.D. Reading Committees

B. Afshari, T. Cheung, R. Henk, U. Piomelli, B. Singer, R. Strawn. K. Yang

Committees

University: Academic Council, Freshman Advisor (1983-1985)  
College of Engineering: Undergraduate Council (1983-1984)  
Mechanical Engineering: Undergraduate Advising Committee (1982-1983),  
Graduate Admissions Committee (1983-1984)

F.2 VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY

Ph.D. Committee: J. Dagenhart

Staff Supervised: R. Eskelund, B. Wong (two computer science students)

Courses Taught: Undergraduate Fluid Mechanics, ESM 3010

G. SCHOLARLY AND CREATIVE CONTRIBUTIONS

G.1 PUBLICATIONS - PAPERS:

1. "Design Considerations of Transonic LFC Airfoils with Practically Full Chord Laminar Flow," with W. Pfenninger, Langley Research Center Basic Research Review, April 1978, p. 20.
2. "Design Considerations of Advanced Supercritical Low Drag Suction Airfoils," with W. Pfenninger and J. R. Dagenhart, Viscous Flow Drag Reduction, AIAA Progress in Astronautics and Aeronautics Series, Vol. 72, 1980.
3. "Flow over Plates with Suction through Porous Strips," with A. H. Nayfeh and S. A. Ragab, AIAA Paper 80-1416, AIAA Journal, Vol. 20, No. 5, May 1982.
4. "A Numerical-Perturbation Technique for Determining the Stability of Flows over Plates with Porous Suction Strips," with A. H. Nayfeh, accepted AIAA Journal (also AIAA Paper 81-1280, "Stability of Flow over Plates with Porous Suction Strips," with A. H. Nayfeh).
5. "The Stability of Boundary Layers with Porous Suction Strips: Experiment and Theory," with G. A. Reynolds, W. S. Saric, and A. H. Nayfeh, prepared for ACEE Project Oral Status Review, Dryden Flight Research Center, September 1981.
6. "Stability of Compressible Three-Dimensional Boundary Layer Flows," with A. H. Nayfeh, AIAA Paper 82-1009.
7. "Stability of Flow over Axisymmetric Bodies with Porous Suction Strips," with A. H. Nayfeh, AIAA Paper 82-1025.
8. "Effects of Suction and Blowing on Boundary-Layer Transition," with W. S. Saric, NASA-CR, 1982, AIAA Paper 83-0043, accepted by AIAA Journal.
9. "Wave Interactions in Swept-Wing Flows," AIAA Paper 84-1678.

10. "An Analysis of Wave Interactions in Swept-Wing Flows," 2nd IUTAM Symposium on Laminar-Turbulent Transition," Novosibirsk, USSR, July 9-13, 1984, to be published by Springer-Verlag.

#### G.2 PRESENTATIONS:

1. "Stability of Flow over Plates with Porous Suction Strips," by H. L. Reed and A. H. Nayfeh, AIAA 14th Fluid and Plasma Dynamics Conference, June 23-25, 1981, Palo Alto, California, AIAA Paper No. 81-1280.
2. "Stability of Compressible Three-Dimensional Boundary-Layer Flows," by H. L. Reed and A. H. Nayfeh, AIAA/ASME 3rd Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference, June 7-11, 1982, St. Louis, Missouri, AIAA Paper No. 82-1009.
3. "Stability of Flow over Axisymmetric Bodies with Porous Suction Strips," by A. H. Nayfeh and H. L. Reed, AIAA/ASME 3rd Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference, June 7-11, 1982, St. Louis, Missouri, AIAA Paper No. 82-1025.
4. "Experiments on the Stability of the Flat-Plate Boundary Layer with Suction," by G. A. Reynolds and W. S. Saric, AIAA/ASME 3rd Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference, June 7-11, 1982, St. Louis, Missouri, AIAA Paper No. 82-1026.
5. "Boundary Layer Stability - Recent Developments," Seminar in Fluid Mechanics, Stanford University, November 2, 1982.
6. "Nonparallel Stability of Compressible Three-Dimensional Boundary-Layer Flows," by H. L. Reed and A. H. Nayfeh, 35th Meeting of the American Physical Society, Division of Fluid Dynamics, Nov. 21-23, 1982, Rutgers, New Jersey, Paper EB4.
7. "Plans for Transition Research," 1983 Meeting of the Stanford Thermosciences Affiliates, Stanford, California, January 24, 1983.
8. "An Analysis of Instabilities in Laminar Viscous Flows," Seminar in Applied Mathematics, Sandia National Laboratories, Albuquerque, NM, August 3, 1983.
9. "The Effect of Suction and Blowing on Boundary Layer Transition," (a) Applied Mechanics Seminar, University of Southern California, February 8, 1984; (b) Fluid Mechanics Seminar, California Institute of Technology, April 25, 1984.
10. "Wave Interactions in Swept-Wing Flows," (a) NASA/Langley Research Center, June 13, 1984; (b) AIAA 17th Fluid Dynamics, Plasma Dynamics, and Lasers Conference, June 25-27, 1984, Snowmass, Colorado, AIAA Paper No. 84-1678; (c) 2nd IUTAM Symposium on Laminar-Turbulent Transition, Novosibirsk, USSR, July 9-13, 1984.